A Closed-form Solution for Options on Assets with Pricing Errors

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Abstract

This paper examines the effects that pricing errors in the underlying asset have on options prices, their greeks, and their implied risk neutral densities. Pricing errors can be viewed as a random proportional transaction cost. When pricing errors are information-unrelated, options prices are unambiguously higher than the Black-Scholes case and increasing in the pricing error variance. Hedging volatility is higher and the optimal exercise price for American put options is decreased. The option implied risk-neutral density and option Greeks are materially affected, which leads to suboptimal risk management and hedging without accounting for the pricing errors.

Keywords: Pricing errors, options prices, mathematical finance

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1. Introduction

Separate large bodies of literature have recently emerged with one examining optimal trading rules (and hedging) with transaction costs (Atmaz and Basak, 2019; Clewlow and Hodges, 1997; Grandits and Schachinger, 2001; Guéant and Pu, 2017; Kabanov and Safarian, 1997; Kallsen and Muhle-Karbe, 2015; Leland, 1985; Lepinette, 2012; Nguyen and Pergamenshchikov, 2017) and with the other identifying and filtering pricing error variances out of observed asset price return variances to recover the uncorrupted volatility matrix (Jacod, Li, and Zheng, 2017; Piccotti, 2020; Zhang, 2006; Zhang, Mykland, and Aït-Sahalia, 2005). Generally, however, these two strings of literature have not been connected to examine the effect that pricing errors in the underlying asset has on options pricing and their associated hedging strategy. I seek to fill this gap in the literature.

Pricing errors can be viewed as a random proportional transaction cost. The seminal work of Leland (1985) shows that an alternative appropriately chosen volatility can be substituted into the Black-Scholes option pricing formula, which accounts for the transaction costs. Further, as an implication of this, transaction costs can be estimated from the observed Black-Scholes prices.[2] I derive a similar result, with respect to pricing error variances, where a suitably chosen volatility can be substituted into the Black-Scholes formula, which accounts for the pricing errors. In this manner, my adjustment to the variance plugged into the standard Black-Scholes formula is similar in nature to that of Leland (1985) to address fixed positive transaction costs and to that of Lo and Wang (1995) to address predictability in the underlying asset price return.

Similar to the case with fixed transactions costs, options prices are higher in my modified Black-Scholes case than in the standard Black-Scholes case to account for the increased cost of

[2] Ofek, Richardson, and Whitelaw (2004) show that violations to put-call parity are strongly related to short sale constraints. As a result, violations to put-call parity could also be used to back out the magnitude of trading costs.

replicating the option payoff. There are two important dissimilarities between pricing errors and transaction costs, however. First, prior studies have generally treated the transaction cost as a constant (function of trading periods), whereas pricing errors are variable. Second, transaction costs are strictly positive, whereas pricing errors can be positive or negative. As a result, in some cases, the trader purchases the asset for cheaper than its true price and in other cases, the trader purchases the asset more expensively than its true price. In this respect, pricing errors represent variable positive and negative transaction costs. Pricing errors in the underlying asset lead to an unhedgable variance, where extant evidence by Gârleanu, Pedersen, and Poteshman (2009) has shown that option prices are affected by demand pressures in an amount proportional to the variance of the unhedgeable part of the option. Pricing errors during trading hours in the underlying asset prices may also contain explanatory power for the finding by Jones and Shemesh (2018) that stock return variance is mispriced in options over weekend periods.

I advance the option pricing literature by deriving closed-form option prices, their respective hedging arguments, and associated greeks, when the underlying asset price contains pricing errors. I assume a linear additive pricing error in log prices, which contains an information-related component and an information-unrelated component. If the information-related component is equal to 0 (there is no correlation between pricing errors and the underlying true asset price innovation), then the call option on an underlying asset with pricing errors is always more expensive than the standard Black-Scholes call option valuation. In the presence of information-related pricing errors, whether a call option price is more or less valuable than the standard Black-Scholes call option price depends critically on the correlation coefficient between pricing error innovations and the true efficient price innovations. This also generally holds for the call option greeks. Specifically, if the correlation is less than minus one-half of the ratio of pricing error volatility to true price return volatility ($\rho \in \left[-\frac{b_\varepsilon}{2b}, 1\right]$, see **Corollary 1**), then the modified call option price is more valuable than the standard Black-Scholes call option price. The percentage pricing errors between the modified call option prices and the standard Black-Scholes options prices are greatest for out-of-the money (OTM) call options, while deep in-the-money (ITM) call options are little affected.

The option greeks are also critically affected when the underlying asset price contains pricing errors. As a result, using the standard Black-Scholes greeks leads to erroneous risk-management and hedging in these cases. In the simple call option case where the pricing error does not contain an information-related component, delta is lower for near ITM strikes and higher for near OTM strikes, gamma is lower for ATM strikes, theta is lower for ATM strikes, vega is lower for ATM strikes and higher both for near ITM and OTM strikes, and rho is lower for near ITM strikes and higher for near OTM strikes. The biases contained in the greeks also depend on the time-to-maturity of the option. Biases in gamma and theta diminish as option time-to-maturity increases, while biases in vega and rho diminish as option time-to-maturity converges to 0. The biases inherent in the option delta are little affected by option time-to-maturity, except for in the case of deep OTM and deep ITM options.

In addition to the standard Black-Scholes first-order greeks, two new greeks exist when the underlying asset contains pricing errors: the option's sensitivity to pricing error volatility (E) and the option's sensitivity to the correlation between the pricing error innovation and the true price innovation (P, the option price's sensitivity to the information-related pricing error component). In both cases, E and P are greatest ATM and dissipate to 0 symmetrically as the option is further ITM or OTM. This pattern suggests that ATM options' notional values are the most effected by pricing errors in the underlying asset (in contrast, percentage errors in the option price relative to the standard Black-Scholes price are greatest for OTM options).

Finally, I extend the base model to its modular representation, which can accommodate stochastic factors in the underlying asset (stochastic volatility, stochastic interest rates, and jumps), to the effects of pricing errors on American option prices, to the effects that pricing errors have on the risk-neutral density, and to the effects that pricing errors have on the value of a firm's equity (priced in a Merton, 1974 model framework).

The remainder of the paper is organized as follows. Section 2 presents the option pricing model. Section 3 extends the model and discusses applications of the model. Section 4 concludes.

2. Model

2.1. Preliminaries

Consider a complete standard financial market $\mathcal{M}$ in a Black-Scholes world.[3] There is a risk-free strictly positive bank account B with its innovations and the innovations in the n'th asset's true price S_n given by:

$$dB(t) = rB(t)dt, \tag{1}$$

$$\frac{dS_n(t)}{S_n(t)} = a_n dt + b_n dW_n(t), \tag{2}$$

[3] $\mathcal{M}$ is standard and complete if (i) it is viable (there are no arbitrage opportunities), (ii) the number of assets in the market N is equal to the dimension of the D-dimensional driving Brownian motions, (iii) the D-dimensional price of risk θ is finite almost surely ($\int_0^T \|\theta(t)\|^2 dt < \infty$), (iv) and the following is a martingale, $Z(t) = \exp\left\{-\int_0^t \theta'(s)ds - \frac{1}{2}\int_0^t \|\theta(s)\|^2 ds\right\}$.

where r is the (constant) risk-free rate, a_n is the (constant) per annum expected return on the asset, b_n is the (constant) per annum standard deviation of the asset's returns, and $dW_n(t)$ is the innovation to a standard Brownian motion. The usual filtration $\mathcal{F}^W = \sigma\{W: 0 \leq s \leq T\}$ is assumed. The true price in unobservable, however, and only the price, which is corrupted by microstructure noise is observable:

$$Y_n(t) = S_n(t)e^{\varepsilon_n(t)}, \tag{3}$$

where $\varepsilon_n(t)$ is the pricing error, which has a distribution dependent on the stochastic differential equation (SDE) that is assumed to govern its evolution $d\varepsilon_n(t)$. Therefore, the financial market described by Equations (1)-(3) differs from the traditional Black-Scholes market only by the presence of the pricing error.

Let the general SDE for the pricing error be:

$$d\varepsilon_n(t) = a_n(\varepsilon_n(t), t)dt + b_{n_\varepsilon} dW_{n_\varepsilon}(t), \tag{4}$$

and let ρ_{n_ε} be the correlation between dS_n and $d\varepsilon_n$. The correlated pricing error structure here is similar in nature to the permanent-transitory pricing error used by Hasbrouck (1993), which is extended to a multi-asset framework by De Jong and Schotman (2010).[4] Using such a mean-reversion model as Equation (4) for the pricing errors is capable of accommodating autocorrelated pricing errors as has been found in Jacod, Li, and Zheng (2017). Note that the volatility parameters in both Equation (2) and Equation (4) can be made constants, even if they are time varying parameters by using the average volatilities over the life of the option (for example, $b_n = \frac{1}{\tau}\int_0^\tau b_n(u)du$ and $b_{n,\varepsilon} = \frac{1}{\tau}\int_0^\tau b_{n,\varepsilon}(u)du$, $b_n(t)$ and $b_{n,\varepsilon}(t)$ can be determined by the solution of

[4] Vanden (2008) also shows that information quality and informed trading affects option prices.

the stochastic differential equation used to describe their evolutions). Therefore, I assume that they are time invariant.

To prevent the pricing error $\varepsilon(t)$ from straying too far away from 0, a mean-reverting stochastic process can be chosen to describe it. For example, suppose that the pricing error is described by a Vasicek (1977) process, $d\varepsilon_n(t) = k_{n_\varepsilon}\left(\zeta_{n_\varepsilon} - \varepsilon(t)\right)dt + b_{n_\varepsilon}dW_{n_\varepsilon}(t)$, which permits both positive and negative pricing errors to arise, where $k_{n_\varepsilon} > 0$ is the mean-reversion intensity coefficient, ζ_{n_ε} is the long-run mean pricing error, b_{n_ε} is the volatility of the pricing error, and $dW_{n_\varepsilon}(t)$ can be correlated with true price's driving Brownian motion $dW_n(t)$ with a correlation coefficient of ρ_{n_ε}. An intuitive example parameterization within this framework for Equation (4) is if $k_{n_\varepsilon} = \frac{1}{dt}$. Then the pricing error fully reverts to 0 each dt-period and is normally distributed around the true price $S_n(t)$ with a mean of 0 and a variance of $\frac{1}{2}b_{n_\varepsilon}^2 dt$.

Note that the multiplicative pricing error term is lognormally distributed, which is similar in form to the Dothan (1978) model and dissimilar to the exponential Vasicek model. The following proposition describes the evolution of the observed price process $\{Y_n(t)\}_{0\leq t\leq T}$.

Proposition 1 *Assume that the true underlying asset price process is described by Equation (2) and that the pricing error process is described by Equation (4). The observed price process* $\{Y_n(t): 0 \leq t \leq T\}$ *evolves according to:*

$$\frac{dY_n(t)}{Y_n(t)} = \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t), t) + \frac{b_{n_\varepsilon}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)dt + b_{n_Y}dW_{n_Y}(t) \tag{5}$$

with the solution

$$Y_n(t) = Y_n(0)\exp\left[\left(a_n + \frac{b_{n_\varepsilon}^2 - b_{n_Y}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)t \right. \qquad (6)$$

$$\left. + b_{n_Y}W_{n_Y}(t) + \int_0^t a_{n_\varepsilon}(\varepsilon_n(u), u)du\right]$$

where $dW_{n_Y}(t) = b_n dW_n(t) + b_{n_\varepsilon}dW_{n_\varepsilon}(t)$, $b_{n_Y}^2 = b_n^2 + b_{n_\varepsilon}^2 + 2b_n b_{n_\varepsilon}\rho_{n_\varepsilon}$.

Proof: See Appendix A.

Proposition 1 shows that the observed price $Y_n(t)$ follows a geometric Brownian motion, which is adjusted by the cumulative compounded effects of pricing errors.

2.2. Hedging in the presence of pricing errors

The method for solving for the hedging strategy follows that of the original Black-Scholes problem. Consider a portfolio that is short a contingent claim and long $H(t)$ units of the underlying asset. Therefore, the problem for the contingent claim writer is to exactly hedge his contingent claim obligation. Precisely, the contingent claim writer wants to find a value $V(0)$ such that when invested in a self-financing trading strategy, yields the contingent claim payoff $V(Y,t)$:

$$0 = -\big(V(Y,t) - V(Y,0)\big) + H_n \bullet Y_n, \qquad (7)$$

where $H_n \bullet Y_n$ denotes the stochastic integral, $H_n \bullet Y_n \stackrel{\text{def}}{=} \int_0^t H_n(u)dY_n(u)$. The SDE corresponding to Equation (7) is:

$$0 = -dV(t) + H_n(t)dY_n(t)$$

$$0 = \left(-V_t - \frac{1}{2}b_{n_Y}^2 Y_n^2(t)V_{Y(t)Y(t)}\right)dt + \big(H_n(t) - V_{Y(t)}\big)dY_n(t)$$

$$0 = \Big(-V_t - \frac{1}{2} b_{n_Y}^2 Y_n^2(t) V_{Y(t)Y(t)} \qquad (8)$$

$$+\big(H_n(t) - V_{Y(t)}\big) Y_n(t) \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t), t) + \frac{b_{n_\varepsilon}^2}{2} + b_n b_{n_\varepsilon} \rho_{n_\varepsilon}\right)\Big) dt$$

$$+ \big(H_n(t) - V_{Y(t)}\big) Y_n(t) b_{n_Y} dW_{n_Y}(t),$$

where $V_t = \frac{\partial V}{\partial t}$, $V_{Y(t)} = \frac{\partial V}{\partial Y_n(t)}$, and $V_{Y(t)Y(t)} = \frac{\partial^2 V}{\partial Y_n^2(t)}$. By choosing $H_n(t) = V_{Y(t)}$, the corresponding portfolio gain process becomes deterministic (risk-free) and as such is required to earn the risk-free rate, in the absence of arbitrage. Setting the deterministic gain of the risk-free terms in Equation (8) equal to the risk-free deterministic gain on the position $-V(Y,t) + V_{Y(t)} Y_n(t)$ gives a modified Black-Scholes equation:

$$V_t + \frac{1}{2} b_{n_Y}^2 Y_n^2(t) V_{Y(t)Y(t)} + r V_{Y(t)} Y_n(t) - r V(Y,t) = 0. \qquad (9)$$

Equation (9) is the Black-Scholes equation with the underlying asset being Y_n in place of S_n and with the variance of the underlying asset increased.

2.3. Contingent claim valuation solution

Consider the Green function approach to solving the PDE in Equation (9). Let $y = \ln Y$ and $V(y,\tau) = e^{-r\tau} w(y,\tau)$, where $\tau = T - t$ is the remaining time-to-maturity for the contingent claim. Substituting y and $e^{-r\tau} w(y,\tau)$ into Equation (9) in place of Y and V, respectively, allows the PDE to be re-written as:

$$\frac{\partial w}{\partial \tau} = \frac{b_{n_Y}^2}{2}\frac{\partial^2 w}{\partial y^2} + \left(r - \frac{b_{n_Y}^2}{2}\right)\frac{\partial w}{\partial y}, \tag{10}$$

where the initial condition is the contingent claim payoff. In the case of a European call option, for example, the initial condition is $w(y,0) = \max(e^y - X, 0)$, where X is the strike price. The infinite dimension Green function of Equation (10) is:

$$\phi(y,\tau;\xi) = \frac{1}{\sqrt{2\pi b_{n_Y}^2 \tau}} \exp\left(- \frac{\left[y + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau - \xi \right]^2}{2 b_{n_Y}^2 \tau} \right), \tag{11}$$

satisfying the initial condition $\lim_{\tau \to 0^+} \phi(y,\tau;\xi) = \delta(y - \xi)$, where $\delta(\cdot)$ is the Dirac delta function representing a unit impulse at the point ξ, and $\tau \to 0^+$ represents converging to 0 from the right. Therefore, the price of a contingent claim V today, by the fundamental theorem of asset pricing, is its discounted expected future payoff:

$$w(y,\tau) = e^{-r\tau} \int_{-\infty}^{\infty} w(\xi,0)\phi(y,\tau;\xi)d\xi, \tag{12}$$

where the payoff of the contingent claim being valued is substituted in for $w(\xi,0)$.

Proposition 2 *Assume that the current observed price equals the true price $Y_n = S_n$. Consider the European call option with a payoff of $c(y,\tau) = max(e^y - X, 0)$. Substituting this payoff into Equation (12) for w and evaluating gives the modified BSM call option price:*

$$\tilde{c}(S_n,\tau) = S_n\, N(\tilde{d}_1) - e^{-r\tau} X\, N(\tilde{d}_2), \tag{13}$$

$$\tilde{p}(S_n,\tau) = e^{-r\tau} X N(\tilde{d}_2) - S_n N(\tilde{d}_1) \tag{14}$$

where $N(x) = \int_{-\infty}^{x} N'(\xi)d\xi$ *denotes the standard normal cumulative distribution function,* $N'(x)$ *is the probability density of the standard normal distribution, and where*

$$\tilde{d}_1 = \frac{\ln\frac{S_n}{X} + \left(r + \frac{b_{n_Y}^2}{2}\right)\tau}{\sqrt{b_{n_Y}^2\tau}}, \quad \tilde{d}_2 = \tilde{d}_1 - \sqrt{b_{n_Y}^2\,\tau}. \tag{15}$$

and where $b_{n_Y}^2 = b_n^2 + b_{n_\varepsilon}^2 + 2b_n b_{n_\varepsilon}\rho_{n_\varepsilon}$.

Proof: See Appendix A.

Proposition 2 suggests that when there are pricing errors in the underlying asset, the BSM option price can be modified by increasing the variance of the underlying asset, which is similar to the method proposed in Leland (1985) to accommodate fixed transactions costs and by Lo and Wang (1995) to accommodate return predictability in the underlying asset. As a result, the hedging volatility is b_{n_Y}. This is the Black-Scholes implied volatility that equates the Black-Scholes price to the option price prevailing with pricing errors in the underlying asset.[5] Using the volatility parameter b_n would lead to either over hedging or under hedging. Whether the European call option price is greater than or less than the Black-Scholes price depends on the correlation ρ_{n_ε} between underlying true asset value changes and pricing error changes as **Corollary 1** outlines.

As is apparent from the call option value's form being the same as the standard Black-Scholes option pricing model, a multiplicative pricing error as considered in Equation (3) cannot explain the implied volatility smile. The implied volatility curve remains flat, but at a higher level than b_n. However, if the underlying asset contains pricing errors, then inverting the Black-Scholes

[5] See Renault and Touzi (1996) for a discussion of the hedging ratio under stochastic volatility.

model gives an implied volatility for the underlying stock, which is erroneous (see Hentschel, 2003), since the resulting volatility estimate also contains the effect of pricing error volatility. Therefore, the primary mechanism through which pricing errors in the underlying asset affect European options trading is through changing the option Greeks, hedging requirements, risk-neutral density. In Section 3.3. , I provide evidence, which shows that pricing errors in the underlying asset also affect the smooth pasting condition for American put options.

Corollary 1 *Assume that the current observed price equals the true price $Y_n = S_n$. Consider the European call option with a payoff of $c(y,\tau) = max(e^y - X, 0)$. Then,*

$$\begin{aligned} \tilde{c} - c^{BS} &\geq 0 \quad for\ \rho_{n_\varepsilon} \in \left[-\frac{b_{n_\varepsilon}}{2b_n}, 1\right] \\ \tilde{c} - c^{BS} &< 0 \quad for\ \rho_{n_\varepsilon} \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right) \end{aligned}. \tag{16}$$

Proof: The option price is a bijective function of variance and $b_{n_Y}^2 < b_n^2$ when $\rho_{n_\varepsilon} \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right)$.

Figure 1 presents the surface of differences between European call option prices when the underlying asset has pricing errors and the respective Black-Scholes European call option prices for the case where $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$. Call prices, when the underlying asset has pricing errors are higher than Black-Scholes prices for options that are near-the-money with the difference in prices and the range of moneyness affected both increasing in time-to-maturity. When $\rho_{n_\varepsilon} < -\frac{b_{n_\varepsilon}}{2b_n}$, the mirror image is observed, with call prices being lower than the Black-Scholes call price. Similarly, a the European put price is relatively more expensive than the Black-Scholes put price when $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$ and vice versa when $\rho_{n_\varepsilon} < -\frac{b_{n_\varepsilon}}{2b_n}$. Therefore, the nature of the information-related pricing errors is critical to determining relative value of options on underlying assets with pricing errors versus the respective Black-Scholes prices. If adverse selection costs are high, $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$ and observed option prices are higher than those implied by the Black-Scholes model. In fact, as long as information frictions and trading barriers are sufficiently small, observed options prices are larger than Black-Scholes call prices.

[Insert Figure 1 about here]

Table 1 presents the modified Black-Scholes call option prices as a function of both the pricing error volatility ($b_{n,\varepsilon}$) and the pricing error innovation's correlation (ρ_{n_ε}) with innovations in the true underlying asset price (S). Call option prices are increasing in both b_{n_ε} as well as in the ρ_{n_ε}, since an increase in each of these variables increases the volatility of the observed underlying asset price (Y) innovations. Following from **Corollary 1**, the call option price is relatively cheap compared to the standard Black-Scholes call option price when $\rho_{n_\varepsilon} \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right)$ and relatively expensive when $\rho_{n_\varepsilon} \in \left[-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$. As the pricing error volatility increases, the modified Black-Scholes option price is higher for all levels of ρ_{n_ε} since $\lim_{\frac{b_n}{b_{n_\varepsilon}} \to 0} \rho_{n_\varepsilon} \in [-1,1] \subset \left[-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$. It is also apparent that the percentage revision in call option price is increasing in moneyness (from ITM to OTM), which shows that very large percentage pricing errors in option prices can arise if the Black-Scholes model is used erroneously, when the underlying asset contains pricing errors.

[Insert Table 1 about here]

The option Greeks associated with the call and put options prices in Equations (13)-(15) are presented in **Proposition 3**.

Proposition 3 *Assume that the current observed price equals the true price $Y_n = S_n$. Consider the European call and put option prices in Equations* (13)-(15). *The associated options greeks are:*

$$\Delta_c = \frac{\partial c}{\partial S_n} = N(\tilde{d}_1), \quad \Delta_p = \frac{\partial p}{\partial S_n} = N(\tilde{d}_1) - 1, \tag{17}$$

$$\Theta_c = -\frac{\partial c}{\partial \tau} = -\frac{S_n b_{n_Y}}{2\sqrt{\tau}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\tilde{d}_1^2}{2}} - rXe^{-r\tau} N(\tilde{d}_2), \tag{18}$$

$$\Theta_p = -\frac{\partial p}{\partial \tau} = -\frac{S_n b_{n_Y}}{2\sqrt{\tau}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\tilde{d}_1^2}{2}} + rXe^{-r\tau} N(-\tilde{d}_2),$$

$$\Gamma = \frac{\partial^2 V}{\partial S_n^2} = \Gamma_c = \Gamma_p = \frac{1}{S_n b_{n_Y} \sqrt{\tau}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\tilde{d}_1^2}{2}} \tag{19}$$

$$v = \frac{\partial V}{\partial b_n} = v_c = v_p = \frac{b_{n_\varepsilon}\rho_{n_\varepsilon} + b_n}{\sqrt{2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon} + b_n^2 + b_{n_\varepsilon}^2}} S_n \frac{1}{\sqrt{2\pi}} e^{-\frac{\tilde{d}_1^2}{2}} \sqrt{\tau} \tag{20}$$

$$\rho_c = \frac{\partial c}{\partial r} = X\tau e^{-r\tau} N(\tilde{d}_2), \quad \rho_p = \frac{\partial p}{\partial r} = -X\tau e^{-r\tau} N(-\tilde{d}_2) \tag{21}$$

$$\mathrm{E}_c = \frac{\partial c}{\partial b_{n_\varepsilon}} = \frac{b_n \rho_{n_\varepsilon} + b_{n_\varepsilon}}{\sqrt{2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon} + b_n^2 + b_{n_\varepsilon}^2}} S_n N'(\tilde{d}_1) \sqrt{\tau} \tag{22}$$

$$\mathrm{P}_c = \frac{\partial c}{\partial \rho_{n_\varepsilon}} = \frac{b_n b_{n_\varepsilon}}{\sqrt{2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon} + b_n^2 + b_{n_\varepsilon}^2}} S_n N'(\tilde{d}_1) \sqrt{\tau} \tag{23}$$

Proof: See Appendix A.

Figure 2 presents the two new Greeks that are introduced, when the underlying asset contains pricing errors and for the case where $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$. E_c, the sensitivity of the call price to the standard deviation of the pricing error, is plotted in Panel (a) and P_c, the sensitivity of the call price to the correlation coefficient of the pricing error with the underlying true price change dS

and the pricing error innovation $d\varepsilon$, is plotted in Panel (b). For both E_c and P_c, the partial derivatives are positive and are the greatest for ATM options. The sensitivities are also positively related to the time-to-maturity across the surface.

[Insert Figure 2 about here]

Figure 3 plots the surfaces of the differences between the European call option Greeks (for the $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$ case), in the presence of pricing errors in the underlying asset, and the Greeks of the Black-Scholes call option price. In Panel (a) and Panel (e), Δ_c and ρ_c, respectively, are decreased for close ITM options and increased for close OTM options. Panels (b) and (d), respectively, show that Γ_c and v_c are lower for ATM options and higher for moderate deviations in moneyness from ATM when the underlying asset has pricing errors versus the Black-Scholes case without pricing errors. Finally, Panel (c) shows that Θ_c is lower for ATM options when the underlying asset has pricing errors versus the Black-Scholes case. Together, the results in Figure 3 provide evidence that pricing errors in the underlying asset have important risk management and hedging implications for options.

[Insert Figure 3 about here]

Table 2-Table 6 present the modified Black-Scholes call option greeks as a function of the underlying asset's pricing error volatility and the pricing error's correlation with the true underlying asset's price innovations. Table 2 contains the call option deltas (Δ_c) from Equation (17). ITM deltas are decreasing in the volatility of the pricing error, while OTM deltas are increasing in it. Likewise, holding the volatility of the pricing error fixed, ITM (OTM) deltas are decreasing (increasing) in the correlation ρ_{n_ε}. Since there is also a bijective relationship between

option delta and variance. $\rho_{n_\varepsilon} = -\frac{b_{n_\varepsilon}}{2b_n}$ is the correlation coefficient value at which point the standard Black-Scholes call option delta is equal to the modified Black-Scholes option delta in Equation (17). As a result, using the standard Black-Scholes delta with $\rho_{n_\varepsilon} > -\frac{b_{n_\varepsilon}}{2b_n}$ causes ITM (OTM) call (put) options to be underhedged and OTM (ITM) call (put) options to be overhedged. The opposite results when $\rho_{n_\varepsilon} < -\frac{b_{n_\varepsilon}}{2b_n}$.

[Insert Table 2 about here]

Table 3 contains the modified call option gammas (Γ_c) from Equation (19). Gammas do not have a monotonic relationship with the volatility of the pricing errors, which is suggested from Panel (b) of Figure 3. While ATM gammas are decreasing in the volatility of the pricing error, ITM gammas and OTM gammas have no monotonic relationship with the pricing error volatility. There is also not a monotonic relationship between option gammas and the correlation coefficient ρ_{n_ε} between the underlying asset's pricing error innovations and underlying true asset price innovations. The following proposition gives the value for ρ_{n_ε}, at which pint Γ is maximized.

Proposition 4 *Consider the option prices given in* **Proposition 2** *and the respective greeks in* **Proposition 3**. *Then, the value of* Γ*, as a function of* ρ_{n_ε}*, reaches its maximum when:*

$$\rho_{n_\varepsilon}^{\max\Gamma} = \frac{-b_n^3 b_{n_\varepsilon} - 2T^{-1} b_n b_{n_\varepsilon} - b_n b_{n_\varepsilon}^3}{2b_n^2 b_{n_\varepsilon}^2} + \frac{2T^{-1}\sqrt{b_n^2 b_{n_\varepsilon}^2 + b_n^2 r^2 T^2 b_{n_\varepsilon}^2 + 2b_n^2 rT \ln\left(\frac{S}{X}\right) b_{n_\varepsilon}^2 + b_n^2 \ln\left(\frac{S}{X}\right)^2 b_{n_\varepsilon}^2}}{2b_n^2 b_{n_\varepsilon}^2}. \quad (24)$$

Proof: Set the partial derivative $\frac{\partial\Gamma}{\partial\rho_{n_\varepsilon}}$ equal to 0 and solve for ρ_{n_ε}.

If $\rho_{n_\varepsilon}^{\max \Gamma} \in [-1,1]$, then there is an interior maximum (which is always the case). Deep ITM and deep OTM call options gammas, however, have a monotonic positive relationship with the correlation coefficient ρ_{n_ε} for $\rho_{n_\varepsilon} \in [-1,1]$ and the maximum is reached at the boundary of $\rho_{n_\varepsilon} = 1$.

[Insert Table 3 about here]

Modified call option thetas are presented in Table 4. Modified call option thetas are negatively related to the volatility of the pricing error as well as negatively related to the correlation coefficient ρ_{n_ε}. The modified call option theta is less than the standard call option theta when $\rho_{n_\varepsilon} > -\frac{b_{n\varepsilon}}{2b_n}$. As a result, market-neutral options strategies (as well as strategies aiming to profit from small market moves) will be placed at the wrong location along the strike price line, if the standard Black-Scholes theta is used, rather than the modified ones in Equation (18).

[Insert Table 4 about here]

Table 5 presents the modified vega from Equation (20). Option vegas when the underlying asset contains pricing errors are negatively related to the pricing error volatility. In fact, when pricing errors are present, the option vega can become negative when there is a sufficiently large negative correlation between the true underlying asset's price innovations and the pricing error innovations. While the relationship between the modified call option vega and the correlation coefficient ρ_{n_ε} is positive for ITM and OTM options, the relationship is a U-shaped pattern for ATM call options, when the pricing error volatility is small (Panel A). The regions for ρ_{n_ε} for at which the modified vega is negative, 0, and positive are given in **Corollary 2**.

Corollary 2 *Consider the option prices given in* **Proposition 2** *and the respective greeks in* **Proposition 3**. *Then,*

$$v < 0 \quad \text{for } \rho_{n_\varepsilon} < -\frac{b_n}{b_{n_\varepsilon}},$$
$$v = 0 \quad \text{for } \rho_{n_\varepsilon} = -\frac{b_n}{b_{n_\varepsilon}},$$
$$v > 0 \quad \text{for } \rho_{n_\varepsilon} > -\frac{b_n}{b_{n_\varepsilon}}. \tag{25}$$

Proof: From Equation (20), $v = 0$ when $b_{n_\varepsilon}\rho_{n_\varepsilon} + b_n = 0$. Solving this for ρ_{n_ε} gives $\rho_{n_\varepsilon} = -\frac{b_n}{b_{n_\varepsilon}}$.

Similar to the other greeks, the modified vega can be greater than or less than the standard Black-Scholes vega. However, there is no analytic solution for the value of ρ_{n_ε}, which sets the modified vega equal to the standard vega.

[Insert Table 5 about here]

Table 6 presents the modified call option rho greek values in Equation (21). As with the Δ_c, Γ, and Θ_c modified call option greeks, the modified call option ρ_c crosses the standard Black-Scholes call option ρ when $\rho_{n_\varepsilon} = -\frac{b_{n_\varepsilon}}{2b_n}$. Whether the modified rho is greater than or less than the standard rho depends on option moneyness, however. For $\rho_{n_\varepsilon} < -\frac{b_{n_\varepsilon}}{2b_n}$, ITM and ATM modified call rhos are greater than the standard Black-Scholes ones, OTM modified call option rhos are less than the standard Black-Scholes ones (see Panel (e) of Figure 3). The moneyness level at which the sign of $\frac{\partial \rho_c}{\partial \rho_{n_\varepsilon}}$ switches sign is also dependent on the volatility of the pricing error, with the level $\frac{X}{S}$ at which this occurs and the pricing error volatility being positively related.

[Insert Table 6 about here]

Table 7 and Table 8 present the two new call option greeks that are unique to the option price when there are pricing errors (E_c and P_c, respectively). Both E_c and P_c are increasing in the volatility of the pricing error and neither E_c nor P_c have a monotonic relationship with the correlation coefficient ρ_{n_ε}. The relationship between E_c and ρ_{n_ε} is a J-pattern and the relation between P_c and ρ_{n_ε} is an inverted J-pattern. While the polynomial forms for both $\frac{\partial \mathrm{E}_c}{\partial \rho_{n_\varepsilon}}$ and $\frac{\partial \mathrm{P}_c}{\partial \rho_{n_\varepsilon}}$ are complicated, each is a cubic function in ρ_{n_ε}, which allows the unique value for ρ_{n_ε} to be solved for, which minimizes (maximizes) E_c (P_c).

[Insert Table 7 and Table 8 about here]

3. Model extensions and applications

In this section, I extend the Black-Scholes framework to show how pricing errors in the underlying asset affect options prices with various exercise policies and payoff structures.

3.1. Risk neutral density

Consider the risk neutral density (RND) for the underlying asset as derived from European call options. It is well known that the RND $f(X)$, in the Black-Scholes framework, for the underlying asset S_n is $F'(X) = f(X) = e^{r\tau}\frac{\partial^2 \tilde{c}(X)}{\partial X^2} = \frac{N'(\tilde{d}_2)}{\sigma\sqrt{\tau}X}$ and that the cumulative density function is $F(X) = 1 - N(\tilde{d}_2)$, where $\tilde{d}_2$ has been substituted in place of d_2 to account for the effect that pricing errors have on options prices (see **Proposition 2**). Figure 4 Panel (a) presents the RNDs derived from Black-Scholes European call option prices for annualized pricing error standard deviations of 0%, 10%, 25%, and 50%. The presence of pricing errors in the underlying asset biases the RND to have a more dispersed distribution. Panel (b) presents the cumulative RND function as a function of the pricing error and underlying asset price innovation correlation coefficient. The RND becomes more dispersed as ρ_{n_ε} increases as well. Panels (c) and (d) plot the respective cumulative RND functions. As a result of the biases created by pricing errors in the underlying asset, out of the money calls and puts are relatively more expensive, when the underlying asset has pricing errors.

[Insert Figure 4 about here]

3.2. General European option pricing model with stochastic factors

The option pricing model can be generalized simply within a modular pricing framework. In this section, I extend the option pricing model to its modular representation, which can accommodate stochastic factors in the true underlying asset, such as stochastic volatility, stochastic interest rates, and jumps. Following the derivations included in Zhu (2010), the general modular form for the European call option price, with pricing errors included is outlined in the following proposition.

Proposition 5 *Assume that the current observed price equals the true price* $Y_n(0) = S_n$*. Consider the European call option with a payoff of* $c(y, \tau) = max(e^y - X, 0)$*. Then,*

$$\tilde{c} = SF_1(\ln Y(T) > \ln X) - XB(0,T)F_2(\ln Y(T) > \ln X), \tag{26}$$

where

$$F_j(\ln Y(T) > \ln X) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \Re\left(f_j(\phi)\frac{e^{-i\phi\ln X}}{i\phi}\right)d\phi \tag{27}$$

for

$$\tilde{f}_j(\phi) = e^{i\phi\ln S}\tilde{f}_j^{SV}(\phi) \times \tilde{f}_j^{SI}(\phi) \times \tilde{f}_j^{PJ}(\phi) \times \tilde{f}_j^{LJ}(\phi),\ j = 1,2. \tag{28}$$

where i solves $m^2 = 1$, $\Re(\cdot)$ *denotes the real part of a complex-valued number, and* $\widetilde{(\cdot)}$ *denotes that* $b_{n,\varepsilon}$ *is used as the volatility parameter in place of* b_n.

Proof: See Appendix A.

$\ln Y(0) = S$, since at time 0, the pricing error is assumed to be equal to 0 and each of the characteristic functions $f_j^a(\phi)$ for $a \in \{SV, SI, PJ, LJ\}$ is with respect to the true underlying asset price S. $f_j^{SV}(\phi)$ is the characteristic function associated with stochastic volatility (example models include Heston, 1993; the double square root process of Longstaff, 1989; Schöbel and Zhu, 1999, among others), $f_j^{SI}(\phi)$ is the characteristic function associated with stochastic interest rates (example models include Cox, Ingersoll, and Ross, 1985; Longstaff, 1989, among others), $f_j^{PJ}(\phi)$ is the characteristic functions associated with Poisson jumps (example models include Cox, Ross, and Rubenstein, 1979; Merton, 1976, among others), and $f_j^{LJ}(\phi)$ is the characteristic function associated with Lévy jumps (example models include Barndorff-Nielsen and Shephard, 2001; Madan, Carr, and Chang, 1998; among others). See Zhu (2010) for the characteristic functions for a variety of different model specifications.

3.3. *American options*

Consider American options on a non-dividend paying underlying asset. Panel (a) of Figure 5 shows that is continues to be suboptimal to exercise an American call option early. Panel (b), however, shows that for American put options, the smooth pasting condition and optimal exercise price are affected by pricing errors in the underlying asset. The optimal exercise boundary is decreasing in the size of the pricing error variance and as an implication of this, the early exercise premium embedded in American options is decreasing in the underlying asset's pricing error variance. This is stated in the following proposition.

Proposition 6 *Assume that the current observed price equals the true price* $Y_n(0) = S_n$ *and that* $\{Y_n(t)\}$ *evolves according to Equation* (5). *The American put option is exercised optimally with a payoff at time* t *of* $\tilde{p}_t(y(t), \tau) = X - e^{y(t)}$ *for* $Y_n(t) \leq Y_n^*(t)$. *Then, in the limits as* $\tau \rightarrow 0^+$ *and as* $\tau \rightarrow \infty$, *it is the case that:*

$$\begin{aligned} Y_n^*(\tau) < S_n^*(\tau), &\quad \rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right] \\ Y_n^*(\tau) = S_n^*(\tau), &\quad \rho_{n_\varepsilon} = -\frac{b_{n_\varepsilon}}{2b_n} \quad , \\ Y_n^*(\tau) > S_n^*(\tau), &\quad \rho_{n_\varepsilon} \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right) \end{aligned} \tag{29}$$

where $S_n^*(t)$ *is the value for the true underlying asset for which the American put option is exercised optimally in the absence of pricing errors; that is,* $p_t(s(t), \tau) = X - e^{s(t)}$ *for* $S_n(t) \leq S_n^*(t)$.

Proof: See Appendix A.

[Insert Figure 5 about here]

3.4. Firm equity value and pricing error volatility

Since a firm's equity value can be viewed as a call option (Merton, 1974) with the firm's face value of defaultable debt as the strike price, the effect that the presence of pricing errors in a firm's stock has on the firm's equity value can be easily examined. Let $D_n > 0$ denote the firm's face value of risky debt (zero-coupon debt so that no coupon payments are made prior to maturity), which is has a term remaining of τ. Denote by A_n the firm's asset value, which has a per annum volatility equal to $b_{n,A}$. If the firm does not make the full payment due at time T (the case where $A_n(T) < D_n$), then the debt holders immediately take over the company and the residual equity claim is worth 0. Note that early optimal default is not allowed in this model. Then, since equity is limited liability, the firm's equity value is $E_n(A_n, T) = \max(A_n(T) - D_n, 0)$.

Proposition 7 *Assume that the current observed price equals the true price $Y_n(0) = S_n$ and that a firm with current assets of A_n has a zero-coupon face value of debt outstanding in the amount of D_n with a term of τ. Then,*

$$\begin{aligned} \tilde{E}_n(A_n, \tau) > E_n(A_n, \tau) \quad &\text{for } \rho \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right] \\ \tilde{E}_n(A_n, \tau) \leq E_n(A_n, \tau) \quad &\text{for } \rho \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right]' \end{aligned} \tag{30}$$

where $\tilde{E}_n$ is the firm's value of equity, when its stock price contains pricing errors, and E_n is the firm's value equity, when its underlying stock price does not contain pricing errors.

Proof: See Appendix A.

Intuitively, **Proposition 7** shows that if the pricing error increases the volatility of the firm's observed equity value ($\rho \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$), which is a transfer of welfare away from the firm's creditors to its shareholders, then the value of the firm's equity is higher. Conversely, a pricing error which is sufficiently negatively correlated with the firm's underlying true stock price ($\rho \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right]$), which transfers welfare away from the firm's shareholders to its creditors, results in a lower equity value for the firm. Merton model probabilities of default ($N(-d_2)$) are presented in Table 9. Consistent with **Proposition 7**, the firm's probability of default in the presence of pricing errors in the underlying stock price is greater than the standard Black-Scholes (Merton, 1974) case, when $\rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$. The probability of default is monotonically increasing in the volatility of the pricing error, since $\frac{\partial b_{n_\varepsilon}}{\partial \rho_{n_\varepsilon}} > 0$. The probability of default, however, is not monotonically increasing in the firm's current equity level ($A_n(0) - D$). This result is due to the relation between the firm's asset volatility, the firm's stock return volatility, equity delta, and debt-to-equity ratio, which given by $b_{n_A} = b_n \left(\frac{\partial E_n}{\partial A_n}\right)^{-1} \frac{E}{V}$ (see Equation (A. 33)).

[Insert Table 9 about here]

While **Proposition 7** follows immediately from option pricing theory, the result also conforms to recent findings in the corporate finance literature with respect to the real effects that stock price efficiency has on firm value. Fang, Noe, and Tice (2009) find that firms' values are increasing in their stock liquidity, which is consistent with **Proposition 7** when the price impact of trade is close to 0; that is, when $\rho \approx 0$. Within a cost of capital context, one possible interpretation for this finding is that a pricing error which makes a firm's stock price more volatile allows the firm to opportunistically raise capital and invest at a lower cost of capital as found in Muñoz (2013).

3.5. Other closed-form option prices

The option pricing arguments presented in Sections 2.1. to 2.3. can be easily extended to options on dividend-paying assets, options on foreign currency, and to exchange options, for example. Assume that the continuously paid constant (for simplicity) yield of an asset is a function of its true price, rather than its observed price, and the dividend process does not contain a pricing error. That is, the wealth process of holding one unit of the asset at its true price is $\hat{S}_n(t) = e^{q_n t} S_n(t)$. Then, in this case, the observed asset price is $Y_n(t) = S_n(t) e^{q_n t + \varepsilon_n(t)}$.

The following 3 cases of options on a dividend paying asset, options on foreign currency,[6] and exchange options can be easily derived utilizing Margrabe's formula[7] (Margrabe, 1978).

Proposition 8 *Assume that the current observed price equals the true price* $Y_n = S_n$ *for* $n = 1,2$. *Consider the European option with a payoff of* $V(y,\tau) = max(\omega[e^y - X], 0)$, *where* $\omega = 1$ *for a call option and* $\omega = -1$ *for a put option. Then,*

$$V(y,\tau,\omega) = e^{-q_1\tau} S_1 N\left(\omega \tilde{d}_1\right)\omega - e^{-q_2\tau} S_2 N\left(\omega \tilde{d}_2\right)\omega, \tag{31}$$

where:

i. $q_2 = r$ *and* $S_2 = X$ *for an option on a dividend-paying asset.*

ii. $q_1 = r$, $q_2 = r^*$, *and* $S_2 = X$ *for an option on foreign currency, where* r *is the domestic risk-free rate and* r^* *is the foreign risk-free rate.*

iii. *As is for an exchange option.*

---

[6] See Garman and Kolhagen (1983) for a derivation of the standard foreign currency option price in the Black-Scholes framework.

[7] Also see Fischer (1978).

$$\tilde{d}_1 = \frac{\ln\frac{S_1}{S_2} + \left(q_1 - q_2 + \frac{b_Y^2}{2}\right)\tau}{\sqrt{b_Y^2 \tau}}, \qquad \tilde{d}_2 = \tilde{d}_1 - b_Y^2\sqrt{\tau},$$

where $b_Y^2 = [1 \quad -1]\begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$,

$$C_{1,1} = b_{1,Y}^2 = b_1^2 + b_{\varepsilon_1}^2 + 2b_1 b_{\varepsilon_1}\rho_{1,\varepsilon_1}, \tag{32}$$

$$C_{1,2} = C_{2,1} = \rho_{1,2}b_1 b_2 + \rho_{\varepsilon_1,\varepsilon_2}b_{\varepsilon_1}b_{\varepsilon_2} + \rho_{1,\varepsilon_2}b_1 b_{\varepsilon_2} + \rho_{2,\varepsilon_1}b_2 b_{\varepsilon_1}, \tag{33}$$

$$C_{2,2} = b_{2,Y}^2 = b_2^2 + b_{\varepsilon_2}^2 + 2b_2 b_{\varepsilon_2}\rho_{2,\varepsilon_2}. \tag{34}$$

Proof: See Appendix A.

4. Conclusion

In this paper, I consider the option pricing problem when the underlying asset contains pricing errors, which are comprised of both information-unrelated and information-related terms. Pricing errors can be viewed as a variable proportional transaction tax, which can be positive (a positive pricing error) or negative (a negative pricing error). As such, the traditional hedging volatility leads to over or under hedging. I derive the closed-form European options prices along with their associated greeks and in this framework, options can be priced correctly by increasing the volatility parameter in the Black-Scholes equation appropriately (**Proposition 2**). This modified hedging volatility in the Black-Scholes equation leads to proper hedging of the European contingent claim. Further, I derive the option sensitivities to the volatility of pricing errors and to the correlation between the underlying asset's true price innovation and pricing error innovations (**Proposition 3**).

When pricing errors are purely information-unrelated (uncorrelated with the true underlying asset value), options prices are unambiguously higher than in the Black-Scholes setting. When underlying asset pricing errors have an information-related term (correlated with the true underlying asset value), the options price is greater than or equal to the Black-Scholes case when the correlation coefficient is sufficiently large (**Corollary 1**). Finally, as model extensions, I show how the risk neutral density is affected when the underlying asset contains pricing errors, how the optimal exercise boundary of American put options is affected (**Proposition 6**), and I derive the general form of the model with stochastic factors (**Proposition 5**), which can accommodate stochastic volatility, stochastic interest rates, Poisson jumps, and Lévy jumps in the true underlying asset price.

Appendix A Proofs

Proof of **Proposition 1**: By Itô's product rule:

$$\begin{aligned} dY_n(t) &= d\left(S_n(t)e^{\varepsilon_n(t)}\right) \\ &= e^{\varepsilon_n(t)}dS_n(t) + S_n(t)d\left(e^{\varepsilon_n(t)}\right) + dS_n(t)d\left(e^{\varepsilon_n(t)}\right). \end{aligned} \tag{A. 1}$$

Apply Itô's lemma to $e^{\varepsilon_n(t)}$:

$$d\left(e^{\varepsilon_n(t)}\right) = e^{\varepsilon_n(t)}\left(a_{n_\varepsilon}(\varepsilon_n(t),t) + \frac{b_{n_\varepsilon}^2}{2}\right)dt + b_{n_\varepsilon}dW_{n_\varepsilon}(t). \tag{A. 2}$$

Then,

$$\begin{aligned} dY_n(t) &= e^{\varepsilon_n(t)}S_n(t)\left(a_n dt + b_n dW_n(t)\right) \\ &\quad + S_n(t)\left(a_n dt + b_n dW_n(t)\right)e^{\varepsilon_n(t)}\left[\left(a_{n_\varepsilon}(\varepsilon_n(t),t) + \frac{b_{n_\varepsilon}^2}{2}\right)dt\right. \end{aligned}$$

$$+b_{n_\varepsilon} dW_{n_\varepsilon}(t)\Big]$$

$$+ S_n(t)\big(a_n dt + b_n dW_n(t)\big)$$

$$\times e^{\varepsilon_n(t)} \left[\left(a_{n_\varepsilon}(\varepsilon_n(t), t) + \frac{b_{n_\varepsilon}^2}{2}\right) dt + b_{n_\varepsilon} dW_{n_\varepsilon}(t)\right]$$

$$\frac{dY_n(t)}{Y_n(t)} = \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t), t) + \frac{b_{n_\varepsilon}^2}{2} + b_n b_{n_\varepsilon} \rho_{n_\varepsilon}\right) dt \qquad \text{(A. 3)}$$

$$+ b_n dW_n(t) + b_{n_\varepsilon} dW_{n_\varepsilon}(t).$$

The new source of randomness $dW_{n_Y}(t) \stackrel{\text{def}}{=} b_n dW_n(t) + b_{n_\varepsilon} dW_{n_\varepsilon}(t)$ can be decomposed into the sum of 2 independent normally distributed random variables by re-writing $dW_{n_\varepsilon}(t)$ as $dW_{n_\varepsilon}(t) \stackrel{\text{def}}{=} \rho_{n_\varepsilon} dW_n(t) + \sqrt{1 - {\rho_{n_\varepsilon}}^2}\, dZ_{n_\varepsilon}(t)$, where $dW_n(t)$ and $dZ_{n_\varepsilon}(t)$ are uncorrelated. Therefore, $dW_{n_Y}(t)$ is also normally distributed with a mean of 0 and a variance of:

$$\mathbb{V}\{dW_{n_Y}(t)\} = \mathbb{E}\left\{\left[b_n dW_n(t) + b_{n_\varepsilon}\left(\rho_{n_\varepsilon} dW_n(t) + \sqrt{1 - {\rho_{n_\varepsilon}}^2}\, Z_{n_\varepsilon}(t)\right)\right]^2\right\}$$

$$= \mathbb{E}\left\{\left[\big(b_n + b_{n_\varepsilon}\rho_{n_\varepsilon}\big) dW_n(t) + b_{n_\varepsilon}\sqrt{1 - \rho_{n_\varepsilon}^2}\, dZ_{n_\varepsilon}\right]^2\right\}$$

$$= \mathbb{E}\big\{b_n^2 + 2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon} + b_{n_\varepsilon}^2 \rho_{n_\varepsilon}^2 + b_{n_\varepsilon}^2\big(1 - \rho_{n_\varepsilon}^2\big)\big\}$$

$$= b_n^2 + 2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon} + b_{n_\varepsilon}^2. \qquad \text{(A. 4)}$$

Therefore, $dW_{n_Y}(t) \stackrel{\text{def}}{=} b_n dW_n(t) + b_{n_\varepsilon} dW_{n_\varepsilon}(t) \sim \mathcal{N}\big(0, b_n^2 + b_{n_\varepsilon}^2 + 2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon}\big)$ and $\frac{dY_n(t)}{Y_n(t)}$ can be re-written as:

$$\frac{dY_n(t)}{Y_n(t)} = \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t), t) + \frac{b_{n_\varepsilon}^2}{2} + b_n b_{n_\varepsilon} \rho_{n_\varepsilon}\right) dt + b_{n_Y} dW_{n_Y}(t), \qquad \text{(A. 5)}$$

where $b_{n_Y}^2 = b_n^2 + b_{n_\varepsilon}^2 + 2 b_n b_{n_\varepsilon} \rho_{n_\varepsilon}$.

Next, consider the solution $Y_n(t)$. Approximate $\frac{dY_n(t)}{Y_n(t)}$ by $d\ln Y_n(t)$. Then, by Itô's lemma,

$$
\begin{aligned}
d\ln Y_n(t) &= \frac{1}{Y_n(t)}dY_n(t) - \frac{1}{2}\cdot\frac{1}{Y_n^2(t)}\big(dY_n(t)\big)^2 \\
&= \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t),t) + \frac{b_{n_\varepsilon}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)dt \\
&\quad + b_{n_Y}dW_{n_Y}(t) - \frac{1}{2}\cdot b_{n_Y}^2 dt \\
&= \left(a_n + a_{n_\varepsilon}(\varepsilon_n(t),t) + \frac{b_{n_\varepsilon}^2 - b_{n_Y}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)dt + b_{n_Y}dW_{n_Y}(t).
\end{aligned}
\tag{A. 6}
$$

Integrating gives:

$$
\begin{aligned}
\ln Y_n(t) &= \ln Y_n(0) + \left(a_n + \frac{b_{n_\varepsilon}^2 - b_{n_Y}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)t + b_{n_Y}W_{n_Y}(t) \\
&\quad + \int_0^t a_{n_\varepsilon}(\varepsilon_n(u),u)du.
\end{aligned}
\tag{A. 7}
$$

Taking the exponential of both sides of Equation (A. 7) yields the solution:

$$
\begin{aligned}
Y_n(t) = Y_n(0)\exp\Bigg[&\left(a_n + \frac{b_{n_\varepsilon}^2 - b_{n_Y}^2}{2} + b_n b_{n_\varepsilon}\rho_{n_\varepsilon}\right)t \\
&+ b_{n_Y}W_{n_Y}(t) + \int_0^t a_{n_\varepsilon}(\varepsilon_n(u),u)du\Bigg]
\end{aligned}
\tag{A. 8}
$$

Notice that when $\varepsilon_n(t) = 0$ for $0 \le t \le T$, $Y_n(t) = S_n(t)$, the solution for a standard geometric Brownian motion.

$Q.E.D.$

Proof of **Proposition 2**: From Equations (11) and (12), the solution for $c(y,\tau)$ is:

$$c(y,\tau) = e^{-r\tau} w(y,\tau)$$

$$= e^{-r\tau} \int_{-\infty}^{\infty} w(\xi, 0) \phi(y, \tau; \xi) d\xi$$

$$= e^{-r\tau} \int_{\ln X}^{\infty} \left(e^{\xi} - X\right) \frac{1}{\sqrt{b_{n_Y} 2\pi}} \exp\left[-\frac{\left(y + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau - \xi\right)^2}{2b_{n_Y}^2 \tau}\right] d\xi$$

$$= e^{-r\tau} \int_{\ln X}^{\infty} e^{\xi} \frac{1}{\sqrt{b_{n_Y} 2\pi}} \exp\left[-\frac{\left(y + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau - \xi\right)^2}{2b_{n_Y}^2 \tau}\right] d\xi$$

$$- e^{-r\tau} X \int_{\ln X}^{\infty} \frac{1}{\sqrt{b_{n_Y} 2\pi}} \exp\left[-\frac{\left(y + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau - \xi\right)^2}{2b_{n_Y}^2 \tau}\right] d\xi$$

$$= e^{-r\tau} e^{y + r\tau} \int_{\ln X}^{\infty} \frac{1}{\sqrt{b_{n_Y} 2\pi}} \exp\left[-\frac{\left(y + \left(r + \frac{b_{n_Y}^2}{2}\right)\tau - \xi\right)^2}{2b_{n_Y}^2 \tau}\right] d\xi$$

$$- e^{-r\tau} X N \left(\frac{y + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau - \ln X}{b_{n_Y}\sqrt{\tau}}\right)$$

$$= S_n N \left(\frac{\ln \frac{S_n}{X} + \left(r + \frac{b_{n_Y}^2}{2}\right)\tau}{2b_{n_Y}^2 \tau}\right) - e^{-r\tau} X N \left(\frac{\ln \frac{S_n}{X} + \left(r - \frac{b_{n_Y}^2}{2}\right)\tau}{b_{n_Y}\sqrt{\tau}}\right),$$

where $y = \ln S_n$, since the pricing error is assumed to 0 at time 0 ($Y_n(0) = S_n(0)e^0 = S_n$)

$Q.E.D.$

Proof of **Proposition 3**: First, the new option greeks (E_c and P_c) and the option vega, which differs materially in form from the standard Black-Scholes option greeks are derived. The derivations for the call option greeks are provided and the put option greeks derivations are omitted, but can be derived by similar methods.

i. $v = \frac{\partial \tilde{c}}{\partial b_n}$

$$\frac{\partial \tilde{c}}{\partial b_n} = \frac{\partial}{\partial b_n}\left[S_n N(\tilde{d}_1) - Xe^{-r\tau}N(\tilde{d}_2)\right]$$

$$= S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1}\frac{\partial \tilde{d}_1}{\partial b_Y}\frac{\partial b_Y}{\partial b_n} - Xe^{-r\tau}\frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2}\frac{\partial \tilde{d}_2}{\partial b_Y}\frac{\partial b_Y}{\partial b_n}$$

$$= \frac{\partial b_Y}{\partial b_n}\left[S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1}\frac{\partial \tilde{d}_1}{\partial b_Y} - Xe^{-r\tau}\frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2}\frac{\partial \tilde{d}_2}{\partial b_Y}\right]$$

$$= \frac{\partial b_Y}{\partial b_n} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

$$= \frac{b_{n_\varepsilon}\rho + b_n}{\sqrt{2b_n b_{n_\varepsilon}\rho + b_n^2 + b_{n_\varepsilon}^2}} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

ii. $E_c = \frac{\partial \tilde{c}}{\partial b_{n_\varepsilon}}$

$$\frac{\partial \tilde{c}}{\partial b_{n_\varepsilon}} = \frac{\partial}{\partial b_{n_\varepsilon}}\left[S_n N(\tilde{d}_1) - Xe^{-r\tau}N(\tilde{d}_2)\right]$$

$$= S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1}\frac{\partial \tilde{d}_1}{\partial b_Y}\frac{\partial b_Y}{\partial b_{n_\varepsilon}} - Xe^{-r\tau}\frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2}\frac{\partial \tilde{d}_2}{\partial b_Y}\frac{\partial b_Y}{\partial b_{n_\varepsilon}}$$

$$= \frac{\partial b_Y}{\partial b_{n_\varepsilon}}\left[S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1}\frac{\partial \tilde{d}_1}{\partial b_Y} - Xe^{-r\tau}\frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2}\frac{\partial \tilde{d}_2}{\partial b_Y}\right]$$

$$= \frac{\partial b_Y}{\partial b_{n_\varepsilon}} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

$$= \frac{b_n \rho + b_{n_\varepsilon}}{\sqrt{2 b_n b_{n_\varepsilon} \rho + b_n^2 + b_{n_\varepsilon}^2}} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

iii. $P_c = \frac{\partial \tilde{c}}{\partial \rho_{n_\varepsilon}}$

$$\frac{\partial \tilde{c}}{\partial \rho_{n_\varepsilon}} = \frac{\partial}{\partial \rho_{n_\varepsilon}} \left[S_n N(\tilde{d}_1) - X e^{-r\tau} N(\tilde{d}_2)\right]$$

$$= S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial b_Y} \frac{\partial b_Y}{\partial \rho_{n_\varepsilon}} - X e^{-r\tau} \frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2} \frac{\partial \tilde{d}_2}{\partial b_Y} \frac{\partial b_Y}{\partial \rho_{n_\varepsilon}}$$

$$= \frac{\partial b_Y}{\partial \rho_{n_\varepsilon}} \left[S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial b_Y} - X e^{-r\tau} \frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2} \frac{\partial \tilde{d}_2}{\partial b_Y}\right]$$

$$= \frac{\partial b_Y}{\partial \rho_{n_\varepsilon}} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

$$= \frac{b_n b_{n_\varepsilon}}{\sqrt{2 b_n b_{n_\varepsilon} \rho + b_n^2 + b_{n_\varepsilon}^2}} S_n N'(\tilde{d}_1)\sqrt{\tau}$$

The remaining option greeks do not differ materially in from from the standard Black-Scholes option greeks. Rather, b_n is replace with b_{n_Y} everywhere in the formulas. For completeness, the derivations are presented (following the derivation procedures in Chen, Lee, and Shih, 2010) regardless.

iv. $\Delta_c = \frac{\partial \tilde{c}}{\partial S_n}$

$$\frac{\partial \tilde{c}}{\partial S_n} = \frac{\partial}{\partial S_n} \left[S_n N(\tilde{d}_1) - X e^{-r\tau} N(\tilde{d}_2)\right]$$

$$= N(\tilde{d}_1) + S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial S_n} - Xe^{-r\tau} \frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2} \frac{\partial \tilde{d}_2}{\partial S_n}$$

$$= N(\tilde{d}_1) + S_n N'(\tilde{d}_1) \frac{1}{S_n b_{n_Y} \sqrt{\tau}} - Xe^{-r\tau} N'(\tilde{d}_1) \frac{S_n}{X} e^{r\tau} \frac{1}{S_n b_{n_Y} \sqrt{\tau}}$$

$$= N(\tilde{d}_1)$$

v. $\Theta_c = -\frac{\partial \tilde{c}}{\partial \tau}$

$$-\frac{\partial \tilde{c}}{\partial \tau} = \frac{\partial}{\partial \tau} \left[-S_n N(\tilde{d}_1) + Xe^{-r\tau} N(\tilde{d}_2) \right]$$

$$= -S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial \tau} + (-r) Xe^{-r\tau} N(\tilde{d}_2) + Xe^{-r\tau} \frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2} \frac{\partial \tilde{d}_2}{\partial \tau}$$

$$= -S_n N'(\tilde{d}_1) \left[\frac{r + \frac{b_{n_Y}^2}{2}}{b_{n_Y} \sqrt{\tau}} - \frac{\ln \frac{S_n}{X}}{2 b_{n_Y} \tau^{\frac{3}{2}}} - \frac{r + \frac{b_{n_Y}^2}{2}}{2 b_{n_Y} \sqrt{\tau}} \right] - rXe^{-r\tau} N(\tilde{d}_2)$$

$$+ Xe^{-r\tau} N'(\tilde{d}_1) \frac{S_n}{X} e^{r\tau} \left[\frac{r}{b_{n_Y} \sqrt{\tau}} - \frac{\ln \frac{S_n}{X}}{2 b_{n_Y} \tau^{\frac{3}{2}}} - \frac{r + \frac{b_{n_Y}^2}{2}}{2 b_{n_Y} \sqrt{\tau}} \right]$$

$$= -S_n N'(\tilde{d}_1) \frac{\frac{b_{n_Y}^2}{2}}{b_{n_Y} \sqrt{\tau}} - rXe^{-r\tau} N(\tilde{d}_2)$$

$$= -\frac{S_n b_{n_Y}}{2\sqrt{\tau}} N'(\tilde{d}_1) - rXe^{-r\tau} N(\tilde{d}_2)$$

vi. $\Gamma_c = \frac{\partial^2 \tilde{c}}{\partial S_n^2}$

$$\frac{\partial^2 \tilde{c}}{\partial S_n^2} = \frac{\partial \Delta_c}{\partial S_n}$$

$$= \frac{\partial}{\partial S_n} N(\tilde{d}_1)$$

$$= \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial S_n}$$

$$= N'(\tilde{d}_1) \frac{\frac{1}{S_n}}{b_{n_Y}\sqrt{\tau}}$$

$$= \frac{1}{S_n b_{n_Y}\sqrt{\tau}} N'(\tilde{d}_1)$$

vii. $\rho_c = \frac{\partial \tilde{c}}{\partial r}$

$$\frac{\partial \tilde{c}}{\partial r} = \frac{\partial}{\partial r}\left[S_n N(\tilde{d}_1) - Xe^{-r\tau} N(\tilde{d}_2)\right]$$

$$= S_n \frac{\partial N(\tilde{d}_1)}{\partial \tilde{d}_1} \frac{\partial \tilde{d}_1}{\partial r} + \tau X e^{-r\tau} N(\tilde{d}_2) - \frac{\partial N(\tilde{d}_2)}{\partial \tilde{d}_2} \frac{\partial \tilde{d}^2}{\partial r} X e^{-r\tau}$$

$$= S_n N'(\tilde{d}_1) \frac{\sqrt{\tau}}{b_{n_Y}} + \tau X e^{-r\tau} N(\tilde{d}_2) - X e^{-r\tau} N'(\tilde{d}_2) \frac{\sqrt{\tau}}{b_{n_Y}}$$

$$= S_n N'(\tilde{d}_1) \frac{\sqrt{\tau}}{b_{n_Y}} + \tau X e^{-r\tau} N(\tilde{d}_2) - X e^{-r\tau} N'(\tilde{d}_1) \frac{S_n}{X} e^{r\tau} \frac{\sqrt{\tau}}{b_{n_Y}}$$

$$= \tau X e^{-r\tau} N(\tilde{d}_2)$$

$Q.E.D.$

Proof of **Proposition 5**: The value today (time 0) of a European call option on Y_n with an expiration date of T, with stochastic interest rates, is:

$$c_0 = \mathbb{E}^{\mathbb{Q}}\left\{e^{-\int_0^T r(t)dt}\left(e^{\ln Y(T)} - e^{\ln X}\right)\cdot \mathbf{1}_{\ln Y(T)>\ln X}\right\} \tag{A. 9}$$

$$= \mathbb{E}^{\mathbb{Q}}\left\{e^{-\int_0^T r(t)dt}e^{\ln Y(T)}\cdot \mathbf{1}_{\ln Y(T)>\ln X}\right\}$$

$$- \mathbb{E}^{\mathbb{Q}}\left\{e^{-\int_0^T r(t)dt}e^{\ln X}\cdot \mathbf{1}_{\ln Y(T)>\ln X}\right\},$$

where $\mathbb{E}^{\mathbb{Q}}\{\cdot\}$ denotes that the expectation is taken under the risk-neutral measure $\mathbb{Q}$. Let Q_1 and Q_2 denote the changes of numeraire associated with the observed underlying asset price $Y(t)$ and the zero-coupon bond price $B(t,T) = \mathbb{E}\left\{\exp\left(-\int_t^T r(u)du\right)\right\}$, respectively. Then the two Radon-Nikodym derivatives are:

$$\frac{dQ_1}{d\mathbb{Q}}(t) = \frac{Y(t)H(0)}{H(t)Y(0)} = e^{-\int_0^t r(u)du}\frac{Y(t)}{Y(0)} = g_1(t), \tag{A. 10}$$

$$\frac{dQ_2}{d\mathbb{Q}}(t) = \frac{B(t,T)H(0)}{H(t)B(0,T)} = e^{-\int_0^t r(u)du}\frac{B(t,T)}{B(0,T)} = g_2(t), \tag{A. 11}$$

where $H(t) = \exp\left(\int_0^t r(u)du\right)$. Since the pricing error factor $e^{\varepsilon(t)}$ is multiplicative, $Y(t)$ is always positive, which makes it a valid numeraire. The call option value at time 0 can be re-written as:

$$c_0 = Y(0)\mathbb{E}^{Q_1}\left\{\mathbf{1}_{\ln Y(T)>\ln X}\right\} - B(0,T)X\mathbb{E}^{Q_2}\left\{\mathbf{1}_{\ln Y(T)>\ln X}\right\}$$

$$= SF_1^{Q_1}\left(\mathbf{1}_{\ln Y(T)>\ln X}\right) - B(0,T)XF_2^{Q_2}\left(\mathbf{1}_{\ln Y(T)>\ln X}\right), \tag{A. 12}$$

where $F_j^{Q_j}, j = 1{,}2$ are standard normal cumulative probability distributions and where $Y(0) = S$. Taking the Fourier transform of probabilities yields the following characteristic functions for $F_j^{Q_j}$:

$$f_j(\phi) = \mathbb{E}^{Q_j}\{e^{i\phi \ln Y(T)}\}, \qquad j = 1,2 \tag{A. 13}$$

$$= \mathbb{E}^{\mathbb{Q}}\{g_j(T)e^{i\phi \ln Y(T)}\}$$

where i is a solution to the equation $x^2 = 1$ (more popularly written as $i = \sqrt{-1}$). The density function for $\ln Y(T)$ is then the inverse Fourier transform of the characteristic function:

$$q_j(\ln Y(T)) = \frac{1}{2\pi} \int_{\mathbb{R}} f_j(\phi) e^{-i\phi \ln Y(T)} d\phi\,, \qquad j = 1,2. \tag{A. 14}$$

Using these density functions, the cumulative density functions are

$$F_j(\ln Y(t) > \ln X) = \int_{\ln X}^{\infty} q_j(\ln Y(T)) d \ln Y(T)$$

$$= \int_{\ln X}^{\infty} \left(\frac{1}{2\pi} \int_{\mathbb{R}} f_j(\phi) e^{-i\phi \ln X} d\phi \right) d \ln Y(T)$$

$$= \frac{1}{2\pi} \int_{\mathbb{R}} f_j(\phi) \left(\int_{\ln X}^{\infty} e^{-i\phi \ln Y(T)} d \ln Y(T) \right) d\phi$$

$$= \frac{1}{2} + \frac{1}{2\pi} \int_{\mathbb{R}} f_j(\phi) \frac{e^{-i\phi \ln X}}{i\phi} d\phi, \tag{A. 15}$$

and the final line, with the appropriate characteristic function $f_j(\phi)$, is substituted into the second line of c_0 above to attain the European call option price.

The characteristic function of a sum of independent random variables is equal to the product of the characteristic functions of each of the random variables. Therefore, to add stochastic factors into the model is as easy as multiplying the characteristic functions for each the stochastic factors and then substituting the resulting characteristic function into Equation (A. 15) and then substituting Equation (A. 15) into Equation (A. 12) (see Chapter 9.2.1 of Zhu, 2010). That is to say:

$$f_j(\phi) = e^{i\phi \ln Y(0)} f_j^{SV}(\phi) \times f_j^{SI}(\phi) \times f_j^{PJ}(\phi) \times f_j^{LJ}(\phi), \qquad j = 1,2,$$

$$f_j(\phi) = e^{i\phi \ln S} f_j^{SV}(\phi) \times f_j^{SI}(\phi) \times f_j^{PJ}(\phi) \times f_j^{LJ}(\phi), \qquad j = 1,2, \qquad \text{(A. 16)}$$

where $\ln Y(0) = \ln S(0)$, since at time 0, the pricing error is assumed to be equal to 0. Each of the characteristic functions $f_j^a(\phi)$ for $a \in \{SV, SI, PJ, LJ\}$ is with respect to the true underlying asset price S.

$Q.E.D.$

Proof of **Proposition 6**: Normalize, the current time t to 0. The value of an American put option is given by its maximum value, if exercised anytime at or prior to maturity. That is,

$$P(Y_n(0), \tau) = \sup_{0 \le t^* \le T} \mathbb{E}^{\mathbb{Q}}\{e^{-rt^*} \max(X - Y_n(0), 0)\}, \qquad \text{(A. 17)}$$

where t^* is the stopping time at which the American put option would be optimally exercises, which is optimized at:

$$t^* = \inf_u\{0 \le u \le T: P(Y_n(u), u) = \max(X - Y_n(u), 0)\}. \qquad \text{(A. 18)}$$

Therefore, the American put option is exercised at the first time its value crosses below the payoff $X - Y_n(t)$ for $Y_n(t) < X$.

Since the holder of the American put option can invest the proceeds in the amount X at the risk-free rate, if exercised early, then the early exercise premium is a function of rX. Then, the current price of an American put option can be decomposed into its European option value plus an early exercise premium (see Myneni, 1992):

$$P(Y_n(0),\tau) = \mathbb{E}^{\mathbb{Q}}\left\{e^{-r\tau}\big(X - Y_n(0)\big)^+\right\} + \mathbb{E}^{\mathbb{Q}}\left\{\int_0^\tau e^{-ru} rX\mathbf{1}_{Y_n(u)<X}\, du\right\}, \tag{A. 19}$$

$$= \tilde{p}(S,\tau) + \tilde{e}(S,\tau),$$

where (recall that $Y_n(0) = S$)

$$\tilde{p}(S,\tau) = \mathbb{E}^{\mathbb{Q}}\{e^{-r\tau}(X - S)^+\}, \tag{A. 20}$$

$$\tilde{e}(S,\tau) = \mathbb{E}^{\mathbb{Q}}\left\{\int_0^\tau e^{-ru} rX\mathbf{1}_{Y_n(u)<X}\, du\right\}. \tag{A. 21}$$

The first term on the r.h.s. is given in **Proposition 2** and is the modified Black-Scholes put option price, while the second term can be re-written as:

$$\tilde{e}(S,\tau) = \int_0^\tau e^{-ru} \int_0^{Y_n^*(u)} rX\psi(Y_n(u); S)dY_n\, du, \tag{A. 22}$$

where ψ is the transition density for Y_n. Evaluating the above integral gives (see Kwok, 2008 ch. 5.2.3):

$$\tilde{e}(S,\tau) = \int_0^\tau e^{-ru}\, rXN\big(-\tilde{d}_{u,2}\big)du, \tag{A. 23}$$

where

$$\tilde{d}_{u,2} = \frac{\ln \dfrac{S}{Y_n^*(\tau - u)} + \left(r - \dfrac{b_{n_Y}^2}{2}\right)u}{b_{n_Y}\sqrt{u}} \tag{A. 24}$$

and u is the time elapsed from the current time. Putting the equations together, gives the analytic price for an American put option on an underlying asset with pricing errors:

$$P(S,\tau) = \tilde{p}(S,\tau) + \int_0^\tau e^{-ru}\, rXN\big(-\tilde{d}_{u,2}\big)du. \tag{A. 25}$$

Applying the boundary condition $P(S^*,\tau) = X - Y_n^*(\tau)$ gives:

$$X - Y_n^*(\tau) = \tilde{p}(S,\tau) + \int_0^{\tau} e^{-ru}\, rXN\big(-\tilde{d}_{u,2}\big)du$$

$$Y_n^*(\tau) = X - \tilde{p}(S,\tau) - \int_0^{\tau} e^{-ru}\, rXN\big(-\tilde{d}_{u,2}\big)du. \qquad \text{(A. 26)}$$

The optimal exercise price $Y_n^*(\tau)$ can be solved numerically by numerically solving the integral representing the early exercise premium. Closed-form solutions for $Y_n^*(\tau)$ do exist in a Black-Scholes world, however, in the limits as $\tau \to 0^+$ and $\tau \to \infty$ (see Evans, Kuske, and Keller, 2002, and Kowk, 2008 ch. 5, respectively) and are:

$$\lim_{\tau \to 0^+} Y_n^*(\tau) = X - Xb_{n_Y}\sqrt{\tau \ln\left(\frac{b_{n_Y}^2}{8\pi\tau r^2}\right)}, \qquad \text{(A. 27)}$$

$$\lim_{\tau \to \infty} Y_n^*(\tau) = \frac{\mu_-}{\mu_- - 1}X, \qquad \text{(A. 28)}$$

where

$$\mu_- = \frac{-\left(r - \frac{b_{n_Y}^2}{2}\right) - \sqrt{\left(r - \frac{b_{n_Y}^2}{2}\right)^2 + 2b_{n_Y}^2 r}}{b_{n_Y}^2}, \qquad \text{(A. 29)}$$

and where b_{n_Y} has been substituted in place of the b_n variable that would prevail in the standard Black-Scholes framework without pricing errors in the underlying asset. Since $\frac{\partial \lim_{\tau \to 0^+} Y_n^*(\tau)}{\partial \rho_{n_\varepsilon}}$ and $\frac{\partial \lim_{\tau \to 0^+} Y_n^*(\tau)}{\partial \rho_{n_\varepsilon}}$ are both negative and $b_n < b_{n_Y}$ for $\rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$, it is the case that:

$$Y_n^*(\tau) < S_n^*(\tau), \quad \rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right] \qquad \text{(A. 30)}$$
$$Y_n^*(\tau) = S_n^*(\tau), \quad \rho_{n_\varepsilon} = -\frac{b_{n_\varepsilon}}{2b_n} \quad .$$
$$Y_n^*(\tau) > S_n^*(\tau), \quad \rho_{n_\varepsilon} \in \left[-1, -\frac{b_{n_\varepsilon}}{2b_n}\right)$$

$Q.E.D.$

Proof of **Proposition 7**: Merton shows that in the standard Black-Scholes framework (in the absence of pricing errors in the firm's underlying stock price), the value of equity is:

$$E_n(A_n, \tau) = A_n N(\hat{d}_1) - D_n e^{-r\tau} N(\hat{d}_2), \qquad \text{(A. 31)}$$

$$\hat{d}_1 = \frac{\ln \frac{A_n}{D_n} + \left(r + \frac{b_{n_A}^2}{2}\right)\tau}{b_{n_A}\sqrt{\tau}}, \qquad \hat{d}_2 = \hat{d}_1 - b_{n_A}\sqrt{\tau}, \qquad \text{(A. 32)}$$

where b_{n_A} is the volatility of the firm's assets, which is unobservable.

Jones, Mason, and Rosenfeld (1984), however, show that in the standard Black-Scholes framework (in the absence of pricing errors in the firm's underlying stock price), the equity volatility and asset volatility are related by $b_n = b_{n_A} \frac{\partial E_n}{\partial A} \frac{A_n(0)}{E_n(0)}$. Re-arranging gives:

$$b_{n_A} = b_n \left(\frac{\partial E_n}{\partial A_n}\right)^{-1} \frac{E_n(0)}{A_n(0)}$$
$$= b_n \frac{1}{N(\hat{d}_1)} \frac{E_n(0)}{A_n(0)}. \qquad \text{(A. 33)}$$

Substitute in b_{n_Y} in place of b_n to get the relationship that holds, when the underlying stock price has pricing errors (see **Proposition 1**) to attain:

$$\tilde{b}_{n_A} = b_{n_Y} \frac{1}{N\left(\tilde{\tilde{d}}_1\right)} \frac{E_n(0)}{A_n(0)}, \tag{A. 34}$$

$$\tilde{\tilde{d}}_1 = \frac{\ln \frac{A_n}{D_n} + \left(r + \frac{\tilde{b}_{n_A}^2}{2}\right)\tau}{\tilde{b}_{n_A}\sqrt{\tau}}. \tag{A. 35}$$

Since $\frac{\partial \tilde{b}_{n_A}}{\partial b_{n_Y}} > 0$ and $b_{n_Y} > b_n$ for $\rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$, it is the case that $\tilde{b}_{n_A} > b_{n_A}$ for $\rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$. Therefore, since $\frac{\partial E_n}{\partial b_{n_A}} > 0$, $\tilde{E}_n - E_n > 0$ for $\rho_{n_\varepsilon} \in \left(-\frac{b_{n_\varepsilon}}{2b_n}, 1\right]$, where the firm's equity value, in the presence of pricing errors in its stock price, is:

$$\tilde{E}_n(A_n, \tau) = A_n N\left(\tilde{\tilde{d}}_1\right) - D_n e^{-r\tau} N\left(\tilde{\tilde{d}}_2\right), \tag{A. 36}$$

$$\tilde{\tilde{d}}_2 = \tilde{\tilde{d}}_1 - \tilde{b}_{n_A}\sqrt{\tau}. \tag{A. 37}$$

$Q.E.D.$

Proof of **Proposition 8**: Let the covariance matrix between observed underlying assets be:

$$\mathbf{C} = \begin{bmatrix} C_{1,1} & C_{1,2} \\ C_{2,1} & C_{2,2} \end{bmatrix} \tag{A. 38}$$

$$C_{1,1} = b_{1,Y}^2 = b_1^2 + b_{\varepsilon_1}^2 + 2b_1 b_{\varepsilon_1} \rho_{1,\varepsilon_1}, \tag{A. 39}$$

$$C_{1,2} = C_{2,1} = \rho_{1,2} b_1 b_2 + \rho_{\varepsilon_1,\varepsilon_2} b_{\varepsilon_1} b_{\varepsilon_2} + \rho_{1,\varepsilon_2} b_1 b_{\varepsilon_2} + \rho_{2,\varepsilon_1} b_2 b_{\varepsilon_1}, \tag{A. 40}$$

$$C_{2,2} = b_{2,Y}^2 = b_2^2 + b_{\varepsilon_2}^2 + 2b_2 b_{\varepsilon_2} \rho_{2,\varepsilon_2} \tag{A. 41}$$

so that

$$d\mathbf{Y}_t = \boldsymbol{\mu}_t \boldsymbol{dt} + \hat{\mathbf{C}} d\mathbf{W}_t, \tag{A. 42}$$

where $d\mathbf{W}_t = (dW_1, dW_2)_t'$ is a vector of independent standard Brownian motion innovations and where

$$\mu_{i,t} = a_i + q_i + a_{i,\varepsilon}(\varepsilon_i(t), t) + \frac{b_{i,\varepsilon_i}^2}{2} + b_i b_{i,\varepsilon_i} \rho_{i,\varepsilon_i}, \qquad i \in \{1,2\} \tag{A. 43}$$

$$\mathbf{C} = \hat{\mathbf{C}}\hat{\mathbf{C}}', \tag{A. 44}$$

where $\hat{\mathbf{C}}$ is lower-triangular.

Consider a portfolio that is short a contingent claim and long $H_n(t)$ units of the underlying assets for $n = \{1,2\}$. Therefore, the problem for the contingent claim writer is to exactly hedge his contingent claim obligation. Precisely, the contingent claim writer wants to find a value $V(0)$ such that when invested in a self-financing trading strategy, yields the contingent claim payoff $V(\mathbf{Y}, t)$, where $\mathbf{Y} = (Y_1, Y_2)'$:

$$0 = -\big(V(\mathbf{Y}, t) - V(\mathbf{Y}, 0)\big) + \mathbf{H} \bullet \mathbf{Y}, \tag{A. 45}$$

where $\mathbf{H} = (H_1, H_2)'$ and $\mathbf{H} \bullet \mathbf{Y} = \sum_{i=1}^{2} \int_0^t H_i(u) dY_i(u)$.

The gradient and Hessian matrix of $V(\mathbf{Y}, t)$ w.r.t. $\mathbf{Y}$ are:

$$D^1 V_{\mathbf{Y}} = \big(V_{Y_1(t)}, V_{Y_2(t)}\big)', \tag{A. 46}$$

$$D^2 V_{\mathbf{Y}} = \begin{pmatrix} V_{Y_1(t)Y_1(t)} & V_{Y_1(t)Y_2(t)} \\ V_{Y_2(t)Y_1(t)} & V_{Y_2(t)Y_2(t)} \end{pmatrix}. \tag{A. 47}$$

The SDE corresponding to Equation (i) is:

$$0 = -dV(t) + \mathbf{H}(t)' d\mathbf{Y}(t) \tag{A. 48}$$

$$0 = -\left(V_t + (D^1 V_{\mathbf{Y}})' d\mathbf{Y}(t) + \frac{1}{2} d\mathbf{Y}(t)'(D^2 V_{\mathbf{Y}}) d\mathbf{Y}(t)\right) + \mathbf{H}(t)' d\mathbf{Y}(t) \tag{A. 49}$$

$$0 = -V_t + (\mathbf{H}(t) - D^1 V_{\mathbf{Y}})' d\mathbf{Y}(t) - \frac{1}{2} d\mathbf{Y}(t)'(D^2 V_{\mathbf{Y}}) d\mathbf{Y}(t). \tag{A. 50}$$

Therefore, setting $\mathbf{H}(t) = \omega D^1 V_{\mathbf{Y}}$, where $\omega = 1$ for a call option and $\omega = -1$ for a put option, makes the portfolio gain in the final line deterministic and the following Black-Scholes equation is attained:

$$V_t + \frac{1}{2}\hat{\mathbf{C}}(D^2 V_{\mathbf{Y}})\hat{\mathbf{C}}' + r(D^1 V_{\boldsymbol{Y}})'\boldsymbol{Y}(t) - rV = 0, \tag{A. 51}$$

which, when imposing the boundary condition $V(\mathbf{Y}, T) = \max\left(0, Y_1(T) - Y_2(T)\right)$ has the Margrabe (1978) solution:

$$V(\mathbf{Y}, \tau, \omega) = e^{-q_1 \tau} S_1 N(\omega d_1)\omega - e^{-q_2 \tau} S_2 N(\omega d_2)\omega. \tag{A. 52}$$

References

Atmaz, Adem and Basak, Suleyman. (2019). Option prices and costly short-selling, *Journal of Financial Markets*, Forthcoming.

Barndorff-Nielsen, Ole E. and Shephard, Neil. (2001). Non-gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics, *Journal of the Royal Statistical Society: Series B* 63, 167-241.

Black, Fischer and Scholes, Myron. (1973). The pricing of options and corporate liabilities, *Journal of Political Economy* 81. 637-654.

Chen, Hong-Yi, Lee, Cheng-Few and Shih, Weikang. (2010). Derivations and applications of Greek letters: review and integration. In: Lee CF., Lee A.C., Lee J. (eds) *Handbook of Quantitative Finance and Risk Management*. Springer, Boston, MA.

Clewlow, Les and Hodges, Stewart. (1997). Optimal delta-hedging under transactions costs, *Journal of Economic Dynamics and Control* 21, 1353-1376.

Cox, John C., Ingersoll, Johnathan E. and Ross, Stephen A. (1985). An intertemporal general equilibrium model of asset prices, *Econometrica* 53, 363-384.

Cox, John C., Ross, Stephen A. and Rubenstein, Mark. (1979). Option pricing: a simplified approach, *Journal of Financial Economics* 7, 229-263.

De Jong, Frank and Schotman, Peter C. (2010). Price discovery in fragmented markets, *Journal of Financial Econometrics* 8, 1-28.

Dothan, L. Uri. (1978). On the term structure of interest rates, *Journal of Financial Economics* 6, 59-69.

Evans, J.D., Kuske, R. and Keller, Joseph B. (2002). American options with dividends near expiry, *Mathematical Finance* 12, 219-237.

Fang, Vivian W., Noe, Thomas H. and Tice, Sheri. (2009). Stock market liquidity and firm value, *Journal of Financial Economics* 94, 150-169.

Fischer, Stanley. (1978). Call option pricing when the exercise price is uncertain, and the valuation of index bonds, *Journal of Finance* 33, 169-176.

Gârleanu, Nicolae, Pedersen, Lasse Heje and Poteshman, Allen M. (2009). Demand-based option pricing, *Review of Financial Studies* 22, 4259-4299.

Garman, Mark B. and Kohlhagen, Steven W. (1983). Foreign currency options, *Journal of International Money and Finance* 2, 231-237.

Grandits, Peter and Schachinger, Werner. (2001). Leland's approach to option pricing: the evolution of a discontinuity, *Mathematical Finance* 11, 347-355.

Guéant, Olivier and Pu, Jiang. (2017). Option pricing and hedging with execution costs and market impact, *Mathematical Finance* 27, 803-831.

Hasbrouck, Joel. (1993). Assessing the quality of a security market: a new approach to transaction-cost measurement, *Review of Financial Studies* 6, 191-212.

Hentschel, Ludger. (2003). Errors in implied volatility estimation, *Journal of Financial and Quantitative Analysis* 38, 779-810.

Heston, Steven L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options, *Review of Financial Studies* 6, 327-343.

Jacod, Jean, Li, Yingying and Zheng, Xinghua. (2017). Statistical properties of microstructure noise, *Econometrica* 85, 1133-1174.

Jones, E. Philip, Mason, Scott P. and Rosenfeld, Eric. (1984). Contingent claims analysis of corporate capital structures: an empirical investigation, *Journal of Finance* 39, 611-625.

Jones, Christopher S. and Shemesh, Joshua. (2018). Option mispricing around nontrading periods, *Journal of Finance* 73, 861-900.

Kabanov, Yuri M. and Safarian, Mher M. (1997). On Leland's strategy of option pricing with transactions costs, *Finance and Stochastics* 1, 239-250.

Kallsen, Jan and Muhle-Karbe, Johannes. (2015). Option pricing and hedging with small transaction costs, *Mathematical Finance* 25, 702-723.

Kwok, Yue-Kuen. (2008). *Mathematical models of financial derivatives*. 2nd Edition. Springer, Berlin, Heidelberg.

Leland, Hayne E. (1985). Option pricing and replication with transaction costs, *Journal of Finance* 40, 1283-1301.

Lepinette, Emmanuel. (2012). Modified leland's strategy for a constant transaction costs rate, *Mathematical Finance* 22, 741-752.

Lo, Andrew W. and Wang, Jiang. (1995). Implementing option pricing models when asset returns are predictable, *Journal of Finance* 50, 87-129.

Longstaff, Francis A. (1989). A nonlinear general equilibrium model of the term structure of interest rates, *Journal of Financial Economics* 23, 195-224.

Madan, Dilip B., Carr, Peter P. and Chang, Eric C. (1998). The variance gamma process and option pricing, *European Financial Review* 2, 79-105.

Margrabe, William. (1978). The value of an option to exchange one asset for another, *Journal of Finance* 33, 177-186.

Merton, Robert C. (1974). On the pricing of corporate debt: the risk structure of interest rates, *Journal of Finance* 29, 449-470.

Merton, Robert C. (1976). Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics* 3, 125-144.

Muñoz, Francisco. (2013). Liquidity and firm investment: evidence for Latin America, *Journal of Empirical Finance* 20, 18-29.

Myneni, Ravi. (1992). The pricing of the American option, *Annals of Applied Probability* 2, 1-23.

Nguyen, Thai Huu and Pergamenshchikov, Serguei. (2017). Approximate hedging problem with transaction costs in stochastic volatility markets, *Mathematical Finance* 27, 832-865.

Ofek, Eli, Richardson, Matthew and Whitelaw, Robert F. (2004). Limited arbitrage and short sale restrictions: evidence from the options markets, *Journal of Financial Economics* 74, 305-342.

Piccotti, Louis R. (2020). A multiscale estimator for pricing errors in high-frequency financial markets, Available at SSRN: https://ssrn.com/abstract=3375054.

Renault, Eric and Touzi, Nizar. (1996). Option hedging and implied volatilities in a stochastic volatility model, *Mathematical Finance* 6, 279-302.

Schöbel, Rainer and Zhu, Jianwei. (1999). Stochastic volatility with an Ornstein-Uhlenbeck process: an extension, *European Finance Review* 3, 23-46.

Vanden, Joel M. (2008). Information quality and options, *Review of Financial Studies* 21, 2635-2676.

Vasicek, Oldrich. (1977). An equilibrium characterization of the term structure, *Journal of Financial Economics* 5, 177-188.

Zhang, Lan. (2006). Efficient estimation of stochastic volatility using noisy observations: a multi-scale approach, *Bernoulli* 12, 1019-1043.

Zhang, Lan, Mykland, Per A. and Aït-Sahalia, Yacine. (2005). A tale of two time scales: determining integrated volatility with noisy high-frequency data, *Journal of the American Statistical Association* 100, 1394-1411.

Zhu, Jianwei. (2010). *Applications of fourier transform to smile modeling: theory and implementation*, Springer, Verlag Berlin Heidelberg.

Table 1

Modified call option price

This table presents modified Black-Scholes call option prices in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option prices when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	21.23	16.45	11.98	8.07	4.98	2.80	1.43	0.66	0.28
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	21.19	16.27	11.34	6.45	2.27	0.33	0.01	0.00	0.00
-0.75	21.19	16.28	11.46	7.06	3.61	1.47	0.47	0.12	0.02
-0.50	21.21	16.36	11.75	7.69	4.49	2.32	1.05	0.42	0.15
-0.25	21.25	16.51	12.10	8.26	5.21	3.02	1.61	0.79	0.36
0.00	21.33	16.70	12.45	8.78	5.82	3.63	2.13	1.18	0.61
0.25	21.43	16.91	12.80	9.25	6.37	4.18	2.61	1.56	0.89
0.50	21.55	17.14	13.14	9.69	6.87	4.68	3.07	1.94	1.18
0.75	21.69	17.37	13.47	10.11	7.33	5.14	3.49	2.30	1.47
1.00	21.84	17.60	13.79	10.50	7.76	5.58	3.90	2.66	1.77
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	21.19	16.28	11.46	7.06	3.61	1.47	0.47	0.12	0.02
-0.75	21.28	16.60	12.27	8.52	5.53	3.34	1.87	0.98	0.48
-0.50	21.55	17.14	13.14	9.69	6.87	4.68	3.07	1.94	1.18
-0.25	21.91	17.72	13.95	10.68	7.97	5.78	4.09	2.83	1.91
0.00	22.31	18.29	14.69	11.56	8.91	6.74	5.01	3.66	2.63
0.25	22.73	18.85	15.38	12.35	9.76	7.61	5.84	4.43	3.32
0.50	23.15	19.39	16.03	13.08	10.54	8.39	6.61	5.16	3.99
0.75	23.56	19.91	16.64	13.75	11.25	9.12	7.33	5.84	4.62
1.00	23.98	20.42	17.22	14.39	11.92	9.80	8.00	6.49	5.23
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	23.15	19.39	16.03	13.08	10.54	8.39	6.61	5.16	3.99
-0.75	23.98	20.42	17.22	14.39	11.92	9.80	8.00	6.49	5.23
-0.50	24.78	21.37	18.29	15.56	13.15	11.05	9.24	7.70	6.38
-0.25	25.54	22.25	19.28	16.62	14.26	12.19	10.38	8.81	7.45
0.00	26.28	23.09	20.20	17.61	15.29	13.24	11.43	9.84	8.46
0.25	26.98	23.88	21.06	18.52	16.24	14.21	12.41	10.82	9.41
0.50	27.65	24.63	21.88	19.39	17.14	15.13	13.33	11.73	10.31
0.75	28.30	25.35	22.65	20.20	17.99	16.00	14.21	12.60	11.17
1.00	28.93	26.04	23.39	20.98	18.79	16.82	15.04	13.44	12.00

Table 2

Modified call option delta

This table presents modified Black-Scholes call option deltas in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option deltas when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	0.99	0.96	0.88	0.75	0.58	0.40	0.24	0.13	0.06
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	1.00	1.00	1.00	0.97	0.67	0.17	0.01	0.00	0.00
-0.75	1.00	0.99	0.96	0.83	0.60	0.33	0.14	0.04	0.01
-0.50	1.00	0.97	0.91	0.78	0.58	0.38	0.21	0.10	0.04
-0.25	0.99	0.95	0.87	0.74	0.58	0.40	0.25	0.14	0.07
0.00	0.97	0.93	0.84	0.72	0.57	0.42	0.29	0.18	0.11
0.25	0.96	0.91	0.82	0.71	0.57	0.43	0.31	0.21	0.13
0.50	0.95	0.89	0.80	0.69	0.57	0.44	0.33	0.23	0.16
0.75	0.94	0.88	0.79	0.69	0.57	0.45	0.34	0.25	0.18
1.00	0.92	0.86	0.78	0.68	0.57	0.46	0.36	0.27	0.20
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	1.00	0.99	0.96	0.83	0.60	0.33	0.14	0.04	0.01
-0.75	0.98	0.94	0.86	0.73	0.57	0.41	0.27	0.16	0.09
-0.50	0.95	0.89	0.80	0.69	0.57	0.44	0.33	0.23	0.16
-0.25	0.92	0.86	0.77	0.68	0.57	0.46	0.36	0.28	0.20
0.00	0.90	0.83	0.75	0.66	0.57	0.48	0.39	0.31	0.24
0.25	0.88	0.81	0.74	0.66	0.57	0.49	0.41	0.33	0.27
0.50	0.86	0.80	0.73	0.65	0.57	0.49	0.42	0.35	0.29
0.75	0.85	0.79	0.72	0.65	0.58	0.50	0.43	0.37	0.31
1.00	0.84	0.78	0.71	0.65	0.58	0.51	0.44	0.38	0.33
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	0.86	0.80	0.73	0.65	0.57	0.49	0.42	0.35	0.29
-0.75	0.84	0.78	0.71	0.65	0.58	0.51	0.44	0.38	0.33
-0.50	0.82	0.76	0.71	0.64	0.58	0.52	0.46	0.41	0.35
-0.25	0.81	0.75	0.70	0.64	0.59	0.53	0.47	0.42	0.38
0.00	0.80	0.75	0.70	0.64	0.59	0.54	0.49	0.44	0.39
0.25	0.79	0.74	0.69	0.64	0.59	0.54	0.50	0.45	0.41
0.50	0.78	0.74	0.69	0.64	0.60	0.55	0.51	0.46	0.42
0.75	0.78	0.73	0.69	0.64	0.60	0.56	0.51	0.47	0.44
1.00	0.77	0.73	0.69	0.65	0.60	0.56	0.52	0.48	0.45

Table 3

Modified call option gamma

This table presents modified Black-Scholes call option gammas in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option gammas when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	0.0027	0.0085	0.0186	0.0299	0.0369	0.0363	0.0294	0.0200	0.0118
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	0.0000	0.0000	0.0003	0.0188	0.1023	0.0727	0.0089	0.0002	0.0000
-0.75	0.0002	0.0022	0.0125	0.0351	0.0547	0.0512	0.0308	0.0127	0.0037
-0.50	0.0015	0.0064	0.0175	0.0321	0.0417	0.0406	0.0307	0.0186	0.0094
-0.25	0.0033	0.0092	0.0188	0.0289	0.0350	0.0346	0.0287	0.0203	0.0126
0.00	0.0048	0.0108	0.0188	0.0264	0.0308	0.0307	0.0267	0.0206	0.0143
0.25	0.0061	0.0117	0.0184	0.0244	0.0278	0.0278	0.0249	0.0203	0.0152
0.50	0.0069	0.0122	0.0179	0.0228	0.0255	0.0256	0.0235	0.0198	0.0156
0.75	0.0076	0.0124	0.0173	0.0214	0.0237	0.0239	0.0222	0.0192	0.0157
1.00	0.0081	0.0124	0.0168	0.0203	0.0222	0.0224	0.0211	0.0187	0.0156
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	0.0002	0.0022	0.0125	0.0351	0.0547	0.0512	0.0308	0.0127	0.0037
-0.75	0.0041	0.0102	0.0189	0.0276	0.0327	0.0325	0.0276	0.0206	0.0136
-0.50	0.0069	0.0122	0.0179	0.0228	0.0255	0.0256	0.0235	0.0198	0.0156
-0.25	0.0082	0.0124	0.0165	0.0198	0.0216	0.0218	0.0206	0.0184	0.0156
0.00	0.0088	0.0122	0.0153	0.0177	0.0191	0.0193	0.0186	0.0171	0.0151
0.25	0.0089	0.0118	0.0143	0.0161	0.0172	0.0175	0.0170	0.0159	0.0144
0.50	0.0089	0.0113	0.0134	0.0149	0.0158	0.0161	0.0158	0.0150	0.0138
0.75	0.0088	0.0109	0.0126	0.0140	0.0147	0.0150	0.0148	0.0142	0.0133
1.00	0.0087	0.0105	0.0120	0.0131	0.0138	0.0141	0.0140	0.0135	0.0127
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	0.0089	0.0113	0.0134	0.0149	0.0158	0.0161	0.0158	0.0150	0.0138
-0.75	0.0087	0.0105	0.0120	0.0131	0.0138	0.0141	0.0140	0.0135	0.0127
-0.50	0.0084	0.0098	0.0110	0.0119	0.0124	0.0127	0.0126	0.0123	0.0118
-0.25	0.0080	0.0092	0.0102	0.0109	0.0114	0.0116	0.0116	0.0114	0.0111
0.00	0.0077	0.0087	0.0095	0.0101	0.0105	0.0108	0.0108	0.0107	0.0104
0.25	0.0073	0.0082	0.0089	0.0095	0.0099	0.0101	0.0101	0.0101	0.0099
0.50	0.0071	0.0078	0.0085	0.0089	0.0093	0.0095	0.0096	0.0095	0.0094
0.75	0.0068	0.0075	0.0080	0.0085	0.0088	0.0090	0.0091	0.0091	0.0090
1.00	0.0066	0.0072	0.0077	0.0081	0.0084	0.0086	0.0087	0.0087	0.0086

Table 4

Modified call option theta

This table presents modified Black-Scholes call option thetas in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option thetas when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	-2.63	-3.33	-4.38	-5.37	-5.73	-5.19	-3.98	-2.62	-1.51
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	-2.36	-2.51	-2.66	-2.95	-3.22	-1.42	-0.15	0.00	0.00
-0.75	-2.37	-2.61	-3.16	-4.05	-4.42	-3.50	-1.93	-0.75	-0.22
-0.50	-2.48	-2.99	-3.91	-4.90	-5.26	-4.61	-3.28	-1.91	-0.94
-0.25	-2.73	-3.51	-4.60	-5.59	-5.95	-5.44	-4.29	-2.95	-1.79
0.00	-3.07	-4.04	-5.22	-6.19	-6.54	-6.14	-5.13	-3.85	-2.63
0.25	-3.45	-4.56	-5.77	-6.72	-7.07	-6.74	-5.84	-4.64	-3.41
0.50	-3.85	-5.05	-6.27	-7.20	-7.56	-7.28	-6.46	-5.34	-4.13
0.75	-4.24	-5.51	-6.74	-7.64	-8.00	-7.77	-7.03	-5.97	-4.80
1.00	-4.64	-5.94	-7.17	-8.05	-8.42	-8.23	-7.55	-6.55	-5.41
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	-2.37	-2.61	-3.16	-4.05	-4.42	-3.50	-1.93	-0.75	-0.22
-0.75	-2.89	-3.78	-4.92	-5.90	-6.26	-5.80	-4.73	-3.42	-2.22
-0.50	-3.85	-5.05	-6.27	-7.20	-7.56	-7.28	-6.46	-5.34	-4.13
-0.25	-4.83	-6.15	-7.38	-8.25	-8.62	-8.44	-7.80	-6.83	-5.71
0.00	-5.74	-7.12	-8.32	-9.16	-9.54	-9.43	-8.91	-8.06	-7.04
0.25	-6.59	-7.97	-9.15	-9.97	-10.36	-10.31	-9.87	-9.13	-8.20
0.50	-7.36	-8.75	-9.90	-10.71	-11.11	-11.11	-10.74	-10.09	-9.23
0.75	-8.08	-9.46	-10.59	-11.39	-11.80	-11.84	-11.53	-10.95	-10.16
1.00	-8.75	-10.12	-11.24	-12.02	-12.45	-12.51	-12.26	-11.74	-11.02
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	-7.36	-8.75	-9.90	-10.71	-11.11	-11.11	-10.74	-10.09	-9.23
-0.75	-8.75	-10.12	-11.24	-12.02	-12.45	-12.51	-12.26	-11.74	-11.02
-0.50	-9.97	-11.31	-12.40	-13.18	-13.63	-13.75	-13.58	-13.17	-12.55
-0.25	-11.05	-12.38	-13.45	-14.22	-14.69	-14.86	-14.76	-14.43	-13.91
0.00	-12.04	-13.35	-14.41	-15.18	-15.67	-15.88	-15.84	-15.58	-15.13
0.25	-12.95	-14.24	-15.29	-16.06	-16.57	-16.82	-16.83	-16.63	-16.25
0.50	-13.79	-15.07	-16.11	-16.89	-17.42	-17.69	-17.75	-17.60	-17.28
0.75	-14.57	-15.85	-16.88	-17.67	-18.21	-18.52	-18.61	-18.51	-18.25
1.00	-15.31	-16.58	-17.61	-18.41	-18.96	-19.30	-19.42	-19.37	-19.15

Table 5

Modified call option vega

This table presents modified Black-Scholes call option vegas in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option vegas when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	2.01	6.35	13.93	22.42	27.68	27.23	22.02	15.01	8.83
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	0.00	0.00	0.08	4.70	25.58	18.17	2.23	0.06	0.00
-0.75	0.06	0.83	4.68	13.18	20.52	19.18	11.55	4.75	1.40
-0.50	0.74	3.22	8.77	16.03	20.87	20.30	15.34	9.32	4.68
-0.25	2.05	5.78	11.75	18.07	21.90	21.64	17.91	12.72	7.90
0.00	3.63	8.13	14.12	19.79	23.09	23.00	19.99	15.44	10.76
0.25	5.29	10.25	16.14	21.34	24.30	24.34	21.81	17.76	13.29
0.50	6.94	12.17	17.91	22.76	25.50	25.63	23.46	19.80	15.57
0.75	8.54	13.92	19.52	24.09	26.66	26.87	24.97	21.63	17.63
1.00	10.07	15.53	20.98	25.34	27.79	28.06	26.38	23.31	19.52
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	-0.09	-1.10	-6.24	-17.57	-27.36	-25.58	-15.40	-6.33	-1.86
-0.75	-0.77	-1.90	-3.54	-5.17	-6.13	-6.09	-5.18	-3.86	-2.56
-0.50	0.87	1.52	2.24	2.85	3.19	3.20	2.93	2.47	1.95
-0.25	3.60	5.43	7.23	8.65	9.45	9.55	9.02	8.04	6.80
0.00	6.58	9.12	11.46	13.26	14.29	14.49	13.93	12.79	11.29
0.25	9.50	12.49	15.14	17.15	18.31	18.60	18.09	16.94	15.35
0.50	12.29	15.56	18.40	20.53	21.79	22.16	21.72	20.62	19.03
0.75	14.92	18.38	21.34	23.55	24.88	25.33	24.97	23.94	22.39
1.00	17.39	20.99	24.02	26.29	27.68	28.20	27.92	26.97	25.49
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	-15.64	-19.81	-23.42	-26.13	-27.73	-28.21	-27.65	-26.24	-24.22
-0.75	-9.78	-11.81	-13.51	-14.79	-15.57	-15.86	-15.71	-15.17	-14.34
-0.50	-4.18	-4.89	-5.49	-5.93	-6.22	-6.34	-6.32	-6.17	-5.92
-0.25	1.00	1.15	1.27	1.36	1.42	1.45	1.45	1.43	1.38
0.00	5.75	6.50	7.12	7.59	7.90	8.07	8.10	8.01	7.82
0.25	10.10	11.30	12.28	13.03	13.55	13.85	13.93	13.83	13.58
0.50	14.12	15.65	16.92	17.89	18.58	18.99	19.14	19.06	18.79
0.75	17.85	19.65	21.13	22.29	23.11	23.63	23.85	23.82	23.56
1.00	21.33	23.33	25.00	26.30	27.25	27.87	28.17	28.19	27.96

Table 6

Modified call option rho

This table presents modified Black-Scholes call option rhos in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option rhos when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	38.85	39.67	38.14	33.52	26.36	18.36	11.32	6.20	3.04
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	39.40	41.87	44.31	45.31	32.40	8.54	0.60	0.01	0.00
-0.75	39.39	41.59	42.21	38.21	28.08	15.72	6.54	2.04	0.48
-0.50	39.16	40.52	39.56	34.91	26.85	17.67	9.90	4.74	1.96
-0.25	38.65	39.24	37.52	32.96	26.16	18.62	11.89	6.83	3.56
0.00	38.00	38.02	35.94	31.63	25.69	19.20	13.22	8.42	4.99
0.25	37.30	36.92	34.68	30.64	25.32	19.57	14.17	9.65	6.20
0.50	36.61	35.95	33.66	29.85	25.03	19.83	14.89	10.63	7.24
0.75	35.94	35.10	32.80	29.21	24.78	20.02	15.46	11.43	8.12
1.00	35.30	34.34	32.06	28.67	24.56	20.17	15.91	12.10	8.89
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	39.39	41.59	42.21	38.21	28.08	15.72	6.54	2.04	0.48
-0.75	38.34	38.61	36.68	32.24	25.91	18.94	12.61	7.68	4.30
-0.50	36.61	35.95	33.66	29.85	25.03	19.83	14.89	10.63	7.24
-0.25	35.00	33.99	31.73	28.43	24.46	20.22	16.11	12.39	9.23
0.00	33.64	32.48	30.35	27.44	24.03	20.42	16.86	13.56	10.65
0.25	32.48	31.28	29.29	26.69	23.68	20.51	17.36	14.39	11.71
0.50	31.49	30.29	28.44	26.08	23.39	20.55	17.72	15.01	12.53
0.75	30.63	29.46	27.73	25.57	23.13	20.55	17.97	15.49	13.17
1.00	29.88	28.75	27.12	25.13	22.89	20.54	18.17	15.87	13.70
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	31.49	30.29	28.44	26.08	23.39	20.55	17.72	15.01	12.53
-0.75	29.88	28.75	27.12	25.13	22.89	20.54	18.17	15.87	13.70
-0.50	28.61	27.56	26.13	24.40	22.48	20.46	18.42	16.41	14.49
-0.25	27.57	26.61	25.33	23.81	22.13	20.36	18.56	16.78	15.06
0.00	26.69	25.81	24.66	23.30	21.81	20.23	18.63	17.03	15.47
0.25	25.94	25.13	24.09	22.87	21.52	20.10	18.65	17.20	15.78
0.50	25.28	24.54	23.58	22.48	21.26	19.97	18.65	17.32	16.02
0.75	24.70	24.01	23.14	22.12	21.01	19.83	18.62	17.40	16.20
1.00	24.17	23.54	22.73	21.80	20.78	19.70	18.58	17.45	16.33

Table 7

Call option E

This table presents modified Black-Scholes call option Es in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option Es when there are no pricing errors in the underlying asset (these are all missing values, since there is no E Greek in the Black-Scholes model) . Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	nan	nan	nan	nan	nan	nan	nan	nan	nan
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	0.00	0.00	-0.08	-4.70	-25.58	-18.17	-2.23	-0.06	0.00
-0.75	-0.01	-0.14	-0.78	-2.20	-3.42	-3.20	-1.92	-0.79	-0.23
-0.50	0.19	0.80	2.19	4.01	5.22	5.07	3.83	2.33	1.17
-0.25	1.02	2.89	5.87	9.03	10.95	10.82	8.95	6.36	3.95
0.00	2.42	5.42	9.42	13.20	15.39	15.33	13.33	10.30	7.17
0.25	4.16	8.05	12.68	16.77	19.09	19.12	17.14	13.95	10.45
0.50	6.08	10.64	15.68	19.92	22.31	22.42	20.52	17.32	13.62
0.75	8.06	13.14	18.43	22.75	25.18	25.37	23.58	20.43	16.65
1.00	10.07	15.53	20.98	25.34	27.79	28.06	26.38	23.31	19.52
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	0.09	1.10	6.24	17.57	27.36	25.58	15.40	6.33	1.86
-0.75	2.82	6.98	12.99	18.96	22.49	22.32	18.99	14.14	9.38
-0.50	6.08	10.64	15.68	19.92	22.31	22.42	20.52	17.32	13.62
-0.25	8.75	13.19	17.55	21.00	22.94	23.19	21.90	19.52	16.53
0.00	10.96	15.20	19.11	22.10	23.82	24.14	23.22	21.32	18.82
0.25	12.86	16.90	20.49	23.20	24.78	25.16	24.47	22.92	20.77
0.50	14.52	18.39	21.75	24.26	25.75	26.19	25.67	24.37	22.49
0.75	16.02	19.74	22.92	25.29	26.73	27.21	26.82	25.71	24.05
1.00	17.39	20.99	24.02	26.29	27.68	28.20	27.92	26.97	25.49
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	15.64	19.81	23.42	26.13	27.73	28.21	27.65	26.24	24.22
-0.75	16.85	20.33	23.27	25.46	26.82	27.32	27.05	26.13	24.69
-0.50	17.75	20.79	23.32	25.21	26.42	26.94	26.85	26.21	25.14
-0.25	18.50	21.22	23.48	25.18	26.30	26.84	26.86	26.42	25.60
0.00	19.15	21.65	23.72	25.29	26.34	26.90	27.01	26.70	26.06
0.25	19.75	22.08	24.00	25.48	26.49	27.07	27.23	27.04	26.53
0.50	20.30	22.50	24.32	25.72	26.71	27.29	27.51	27.40	27.01
0.75	20.83	22.92	24.65	26.00	26.97	27.57	27.83	27.79	27.48
1.00	21.33	23.33	25.00	26.30	27.25	27.87	28.17	28.19	27.96

Table 8

Call option P

This table presents modified Black-Scholes call option Ps in Panel B-D when the underlying asset contains pricing errors. Panel A contains the baseline Black-Scholes call option Ps when there are no pricing errors in the underlying asset (these are all missing values, since there is no P Greek in the Black-Scholes model) . Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $S = 100$, $b_n = 0.15$, $T = 0.5$, $r = 0.03$.

	K								
$\rho_{n,\varepsilon}$	80	85	90	95	100	105	110	115	120
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)									
BSC	nan	nan	nan	nan	nan	nan	nan	nan	nan
Panel B: $b_{n,\varepsilon} = 0.10$									
-1.00	0.00	0.00	0.02	1.41	7.68	5.45	0.67	0.02	0.00
-0.75	0.01	0.17	0.94	2.64	4.10	3.84	2.31	0.95	0.28
-0.50	0.11	0.48	1.31	2.40	3.13	3.04	2.30	1.40	0.70
-0.25	0.25	0.69	1.41	2.17	2.63	2.60	2.15	1.53	0.95
0.00	0.36	0.81	1.41	1.98	2.31	2.30	2.00	1.54	1.08
0.25	0.45	0.88	1.38	1.83	2.08	2.09	1.87	1.52	1.14
0.50	0.52	0.91	1.34	1.71	1.91	1.92	1.76	1.48	1.17
0.75	0.57	0.93	1.30	1.61	1.78	1.79	1.66	1.44	1.18
1.00	0.60	0.93	1.26	1.52	1.67	1.68	1.58	1.40	1.17
Panel C: $b_{n,\varepsilon} = 0.25$									
-1.00	0.03	0.41	2.34	6.59	10.26	9.59	5.77	2.37	0.70
-0.75	0.77	1.90	3.54	5.17	6.13	6.09	5.18	3.86	2.56
-0.50	1.30	2.28	3.36	4.27	4.78	4.80	4.40	3.71	2.92
-0.25	1.54	2.33	3.10	3.71	4.05	4.09	3.87	3.44	2.92
0.00	1.64	2.28	2.87	3.32	3.57	3.62	3.48	3.20	2.82
0.25	1.68	2.20	2.67	3.03	3.23	3.28	3.19	2.99	2.71
0.50	1.68	2.12	2.51	2.80	2.97	3.02	2.96	2.81	2.59
0.75	1.66	2.04	2.37	2.62	2.76	2.81	2.77	2.66	2.49
1.00	1.63	1.97	2.25	2.46	2.60	2.64	2.62	2.53	2.39
Panel D: $b_{n,\varepsilon} = 0.50$									
-1.00	3.35	4.24	5.02	5.60	5.94	6.04	5.92	5.62	5.19
-0.75	3.26	3.94	4.50	4.93	5.19	5.29	5.24	5.06	4.78
-0.50	3.13	3.67	4.11	4.45	4.66	4.75	4.74	4.63	4.44
-0.25	3.00	3.44	3.81	4.08	4.26	4.35	4.36	4.28	4.15
0.00	2.87	3.25	3.56	3.79	3.95	4.04	4.05	4.01	3.91
0.25	2.76	3.08	3.35	3.55	3.70	3.78	3.80	3.77	3.70
0.50	2.65	2.94	3.17	3.35	3.48	3.56	3.59	3.57	3.52
0.75	2.55	2.81	3.02	3.18	3.30	3.38	3.41	3.40	3.37
1.00	2.46	2.69	2.88	3.04	3.14	3.22	3.25	3.25	3.23

Table 9

Probability of default

This table presents modified Merton (1974) model probabilities of default ($N(-d_2)$) in Panel B-D when the underlying asset contains pricing errors. Probabilities are presented in basis points, where `1 bp $= \frac{1\%}{100}$. Panel A contains the baseline Merton model probabilities of default when there are no pricing errors in the underlying asset. Panels B, C, and D respectively consider annualized pricing error volatilities of $b_{n,\varepsilon} = 0.10$, $b_{n,\varepsilon} = 0.25$, and $b_{n,\varepsilon} = 0.50$, respectively. $\rho_{n,\varepsilon}$ is the correlation between innovations in underlying asset price and its pricing error ($\rho_{n,\varepsilon} = \frac{\mathbb{CV}\{dS,d\varepsilon\}}{\sqrt{\mathbb{V}\{dS\}}\sqrt{\mathbb{V}\{d\varepsilon\}}}$). The remaining pricing parameters are: $A = 100$, $b_n = 0.45$, $T = 0.5$, $r = 0.03$, $b_{n,A} = \frac{b_n}{N(\hat{d}_1)}\frac{(A-D)}{V}$, $\tilde{b}_{n,A} = \frac{b_n}{N(\tilde{\hat{d}}_1)}\frac{(A-D)}{V}$.

	D			
$\rho_{n,\varepsilon}$	80	85	90	95
Panel A: Black-Scholes ($b_{n,\varepsilon} = 0$)				
BSC	12.562	24.950	41.186	38.111
Panel B: $b_{n,\varepsilon} = 0.10$				
-1.00	0.194	0.719	2.087	2.459
-0.75	1.040	3.011	6.921	7.310
-0.50	3.441	8.315	16.260	16.037
-0.25	8.394	17.720	30.813	29.047
0.00	16.715	31.800	50.618	46.263
0.25	28.897	50.637	75.268	67.312
0.50	45.099	73.964	104.128	91.678
0.75	65.220	101.304	136.487	118.805
1.00	88.979	132.094	171.649	148.160
Panel C: $b_{n,\varepsilon} = 0.25$				
-1.00	0.000	0.000	0.000	0.000
-0.75	0.009	0.052	0.233	0.346
-0.50	1.618	4.381	9.483	9.756
-0.25	15.069	29.120	46.965	43.115
0.00	51.382	82.658	114.561	100.441
0.25	111.284	159.966	202.632	173.973
0.50	189.298	252.423	301.557	256.337
0.75	279.010	352.778	404.630	342.370
1.00	375.157	456.010	507.724	428.826
Panel D: $b_{n,\varepsilon} = 0.50$				
-1.00	0.000	0.000	0.000	0.000
-0.75	0.094	0.388	1.248	1.546
-0.50	24.391	43.841	66.549	59.900
-0.25	135.605	189.506	234.794	200.748
0.00	310.560	387.058	439.138	371.259
0.25	507.012	592.816	641.284	541.555
0.50	702.205	788.658	828.438	700.995
0.75	886.766	968.907	997.880	846.847
1.00	1057.679	1132.831	1150.360	979.291

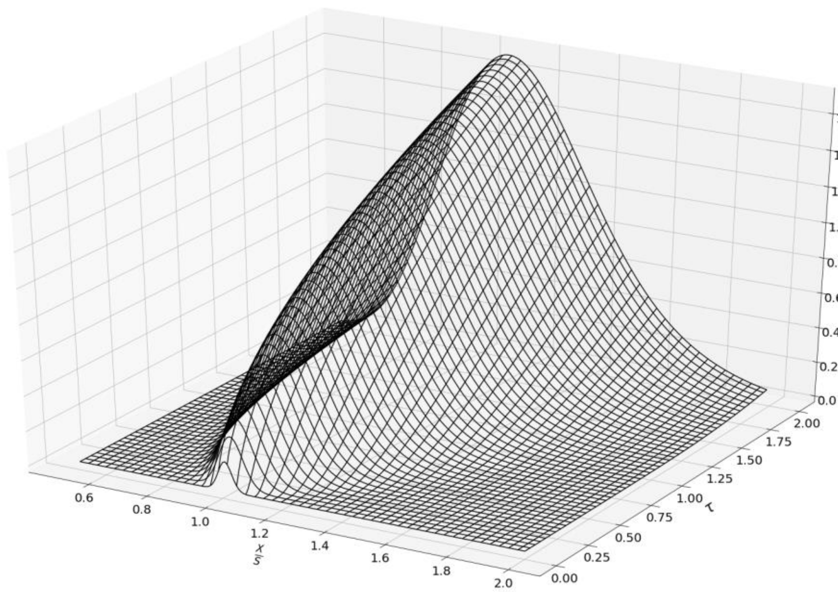
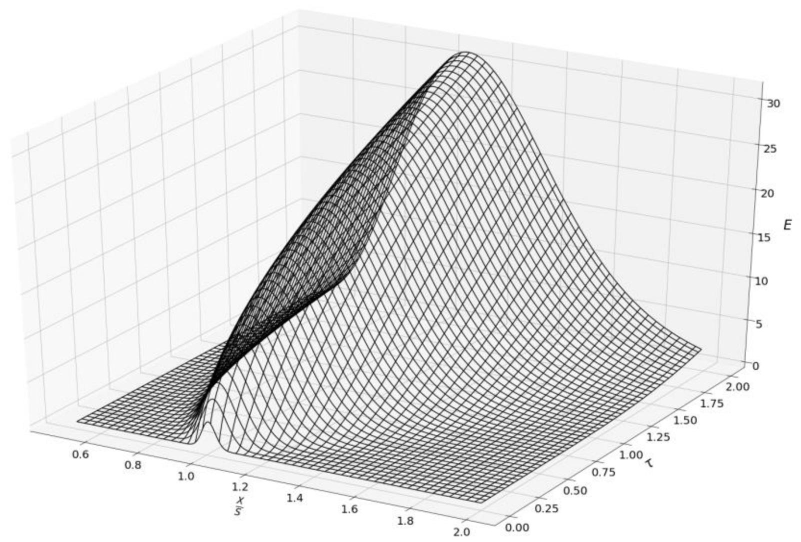


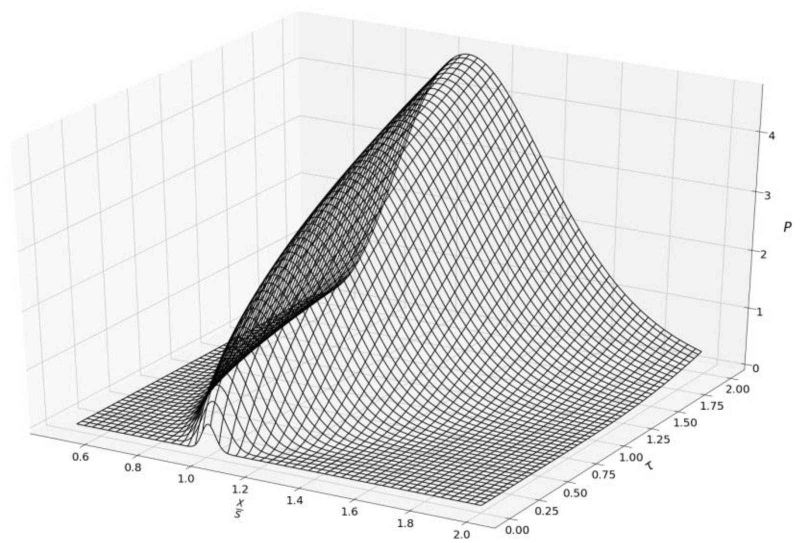
Figure 1

European call price difference

This figure plots the surface of the difference $\tilde{c} - c^{BS}$ between the European call option price with pricing errors in the underlying asset and the Black-Scholes European call option price. The parameters used for the figure are: $S(t) = 100$, $r = 0.03$, $b = 0.15$, $b_\varepsilon = 0.10$, and $\rho_\varepsilon = 0$. $\frac{X}{S}$ denotes option moneyness and $\tau = T - t$ denotes the time-to-maturity in years.



(a)



(b)

Figure 2

E_c and P_c Greeks

This figure plots the European call option price sensitivity to the pricing error volatility $E_c = \frac{\partial c}{\partial b_\varepsilon}$ in Panel (a) and the European call option price sensitivity to the correlation between pricing error innovations and underlying asset true price innovations $P_c = \frac{\partial c}{\partial \rho_\varepsilon}$ in Panel (b). The parameters used for the figure are: $S(t) = 100$, $r = 0.03$, $b = 0.15$, $b_\varepsilon = 0.10$, and $\rho_\varepsilon = 0$. $\frac{X}{S}$ denotes option moneyness and $\tau = T - t$ denotes the time-to-maturity in years.

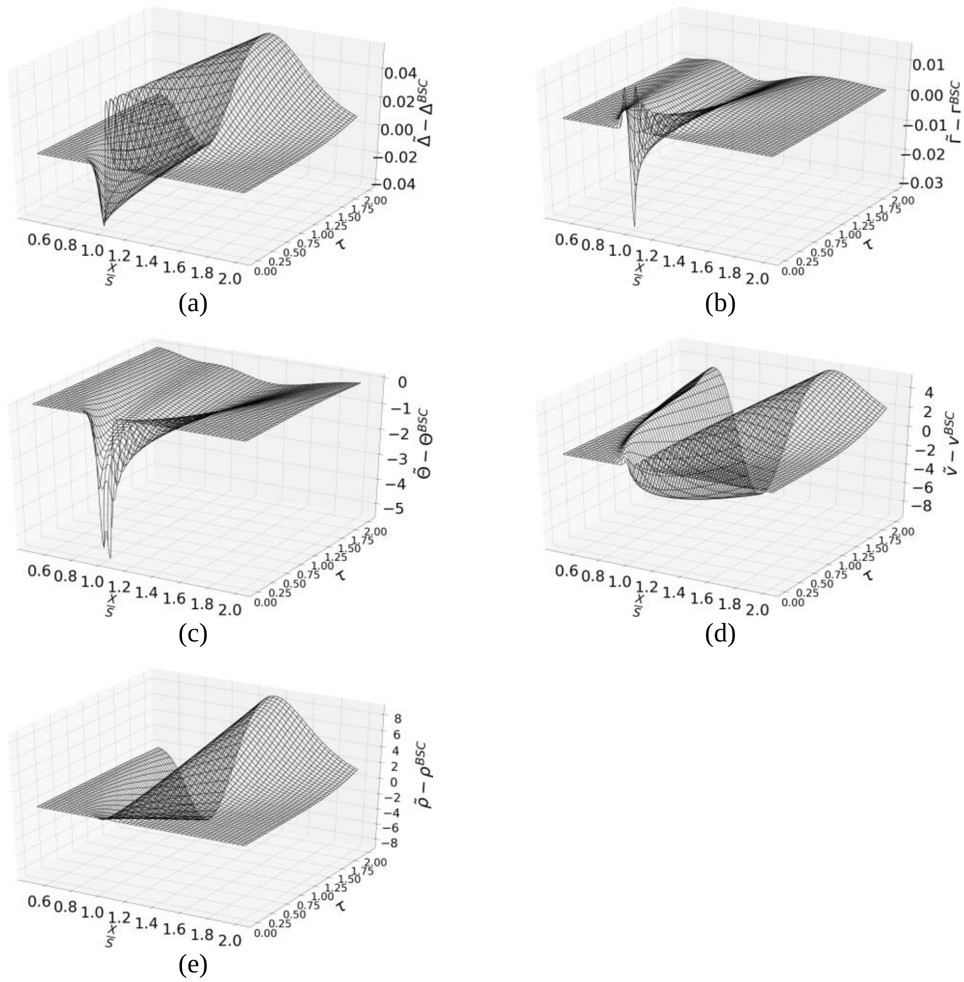


Figure 3

Option Greek differences

This figures presents the difference between the modified European call option greeks when the underlying asset has pricing errors and standard Black-Scholes European call option greeks. Δ is in Panel (a), Γ is in Panel (b), Θ is in Panel (c), v is in Panel (d), and ρ is in Panel (e). Definitions of the greeks are provided in **Proposition 3**. The parameters used for the figure are: $S(t) = 100$, $r = 0.03$, $b = 0.15$, $b_\varepsilon = 0.10$, and $\rho_\varepsilon = 0$. $\frac{X}{S}$ denotes option moneyness and $\tau = T - t$ denotes the time-to-maturity in years.

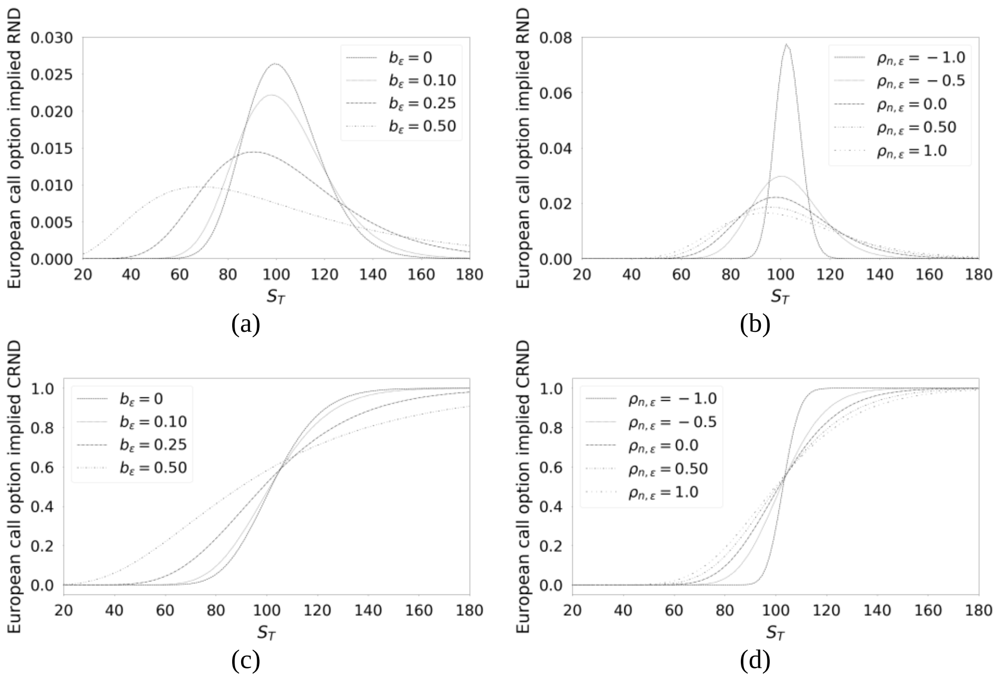


Figure 4

Risk neutral density comparative statics

This figure presents the European call option implied risk neutral probability density (RND) as a function of the underlying asset's pricing error volatility in Panel (a) and the associated risk neutral cumulative probability density (CRND) in Panel (c). The European call option implied risk neutral probability density (RND) as a function of the correlation between pricing errors and underlying asset innovations ($\rho_{n,\varepsilon}$) is plotted in Panel (b) and the associated risk neutral cumulative probability density (CRND) in Panel (d). The parameters used for the figure are: $S(t) = 100$, $r = 0.03$, $b = 0.15$, $T = 1$, $b_\varepsilon \in \{0, 0.10, 0.25, 0.50\}$. $\rho_{n,\varepsilon} = 0$ in Panels (a) and (c). S_T denotes the terminal stock price at expiration. The RND and CRND are, respectively, calculated as:

$$RND(X) = \frac{N'(\tilde{d}_2)}{b_Y\sqrt{\tau}X}, \qquad CRND(X) = 1 - N(\tilde{d}_2), \qquad \tilde{d}_2 = \frac{\ln\frac{S}{X} + \left(r - \frac{b_Y^2}{2}\right)\tau}{\sqrt{b_Y^2\tau}},$$

where $b_Y^2 = b^2 + b_\varepsilon^2 + \rho_{n,\varepsilon} b_n b_{n,\varepsilon}$. Note the differing y-axes in Panels (a) and (b).

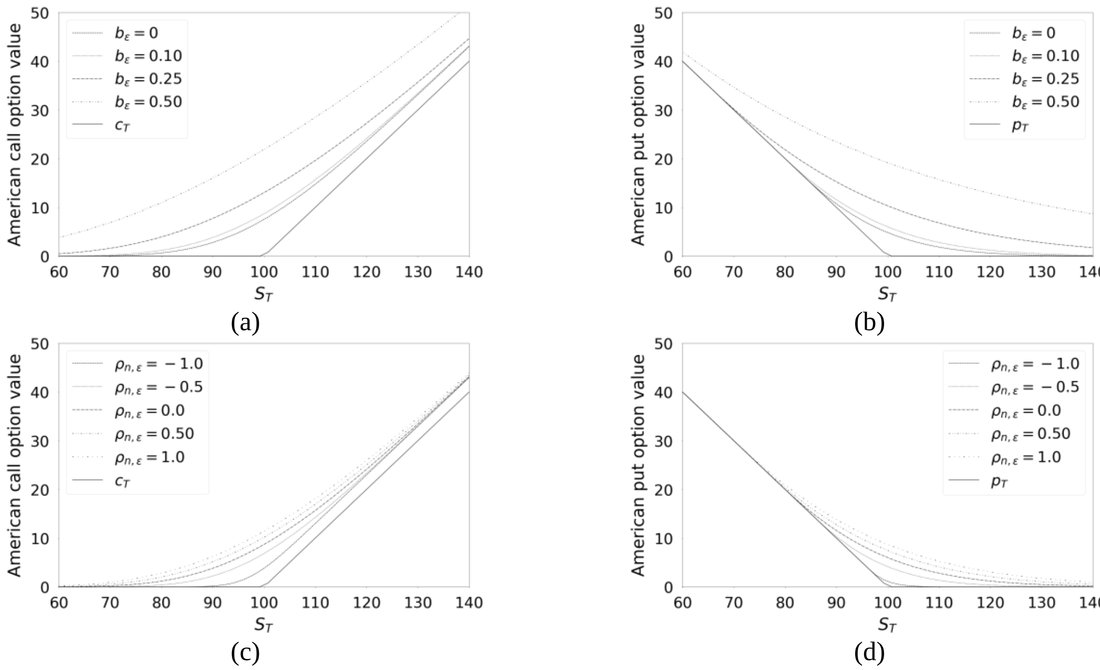


Figure 5

American option price comparative statics

This figure plots the American option prices when the underlying asset contains pricing errors. As a function of the underlying asset pricing error volatility (b_ε), American call option values are plotted in Panel (a) and American put option values are plotted in Panel (b). As a function of the correlation between pricing errors and underlying asset price innovations ($\rho_{n,\varepsilon}$), American call option values are plotted in Panel (c) and American put option values are plotted in Panel (d) The parameters used for the figure are: $S(t) = 100$, $X = 100$, $r = 0.03$, $b = 0.15$, $T = 1$, $b_\varepsilon \in \{0, 0.10, 0.25, 0.50\}$. $\rho_{n,\varepsilon} = 0$ in Panels (a) and (b). S_T denotes the terminal stock price at expiration. Note the different x-axis (S_T) in Panel (d).